GRAPH THEORY APPLIATIONS ON AGRO-ECOSYSTEM

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ABSTRACT

An Agro-ecosystem is always directed by the population interactions living in it and also depends on many aboitic factors. There will be a direct or indirect influence of the elements of the ecosystem. Graph theory is used in different areas which provides a powerful tool to model these direct and indirect interactions. The Effect graph is constructed for the Agro-ecosystem taking vertices as the elements of ecosystem and connected by the edges that correspond to the interactions between them with the weights' representing how strongly these elements effect one another. Using Graph theory concepts some of the cases when one of the factors is eliminated from the ecosystem and if there is an undesirable increase of amount of one of the factors of the ecosystem are discussed by Effect graph. To find the cheapest and fastest possible ways of increasing the yield by Floyd's algorithm which gives the shortest path for a graph structure.

KEYWORDS: *Agro-ecosystem, Effect Graph, Floyd's algorithm*

1. INTRODUCTION

Agro-ecosystems, are defined as communities of plants and animals interacting with their physical and chemical environments that have been modified by people to produce food, fibre, fuel and other products for human consumption and processing. An Agro-ecosystem is very complex system and several different models are introduced to investigate it. Interactions between food web and structure of a food web are studied [7][8] [5].Soil-plant-weather models are also constructed and were examined[1] [4] 6]. Agricultural production meeting several requirements such as " precision and sustainability " become more and more in focus. The aim of the paper is to design a model that describes the interaction process such as tracking and tracing the effects of an element in an agro-ecosystem. Graph theory is used in different areas of mathematics, physics, chemistry, molecular biology, economy and other sciences. [6] [9] [10] and in agriculture [4] [6]. Several indirect and hidden types of interactions which cannot be directly expressed using graph theory. For a given problem to describe the situation and find the solution we need to translate the question in to mathematical language[2][3]. In 1735, the problem of Koinsberg bridge is the origin of graph theory. The town of Koinsberg in Prussia was divided in to four sections included the two regions of the Pregel, Kneiph island and the region between the two branches of the pregel. The swiss mathematician Leonard Euler solved this problem which is the first use of graph theory. Graphs that have number assigned to each edge are called weighted graphs which are widely used in modelling computer networks[3].

In section 2 basic definations of graph theory are given

In section 3 Effect graph will be constructed for the elements of an agro-ecosystem for the growth of a plant[11]. Analyse the effect graph when one of the factor is eliminated from the system.

In section 4 Effect graph is analysed when one of the factor is undesirably increased in the system.

In section 5 Floyd's Algorithm is discussed to find the cheapest the fastest possible ways to increase the yield.

2. GRAPH THEORY

A graph G = (V, E) consists of V, a nonempty set of *vertices (or nodes)* and E, a set of *edges*. Each edge has either one or two vertices associated with it which are called end points. An edge is said to *connect* its end points. A graph in which each edge connects two different vertices and where no two edges connect the same pair of vertices is called a *simple graph.* A *directed graph(or digraph*) consists of a nonempty set of vertices V and a set of directed edges E. The *undirected graphs* consists of set of undirected edges. Graphs that have a weight assigned to each edge are called *weighted graphs*. A *path* is a sequence of edges that begins at a vertex to vertex along the edges of the graph. The matrix formed by the weighted edges represented in rows and columns is called *incidence matrix.*

3. EFFECT GRAPH

First, the graph of our agro-ecosystem is constructed. The vertices of the graph are the elements of the agro-ecosystem. For example in a very simple food web the cultivated plant is our central element, and the others can be the elements that effect the growth of the plant like weather conditions, temperature, watering. Elements of our extended system can also be soil, weather conditions like temperature or precipitation, agro techniques (watering, fertilization). For describing the interactions between the elements of the system we use the edges of the graph. We put an edge between two elements if there is a relationship between them. We direct the edges form an element to the other one if the certain element has an

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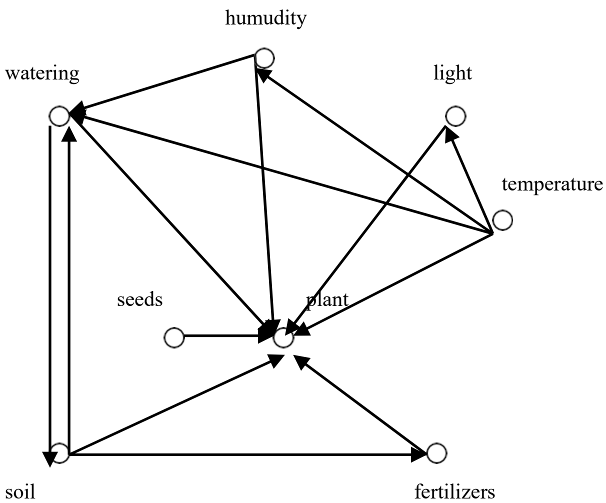
effect on the other element. We allow to have edges between two elements in both directions. Every edge will get a weight showing how strongly the elements effect one another. The weight is a number that expresses the influence of one element to the other one and it is positive or negative depending on whether this effect acts positively or negatively on the element. On the graph theory language this way we obtain a weighted directed graph. This

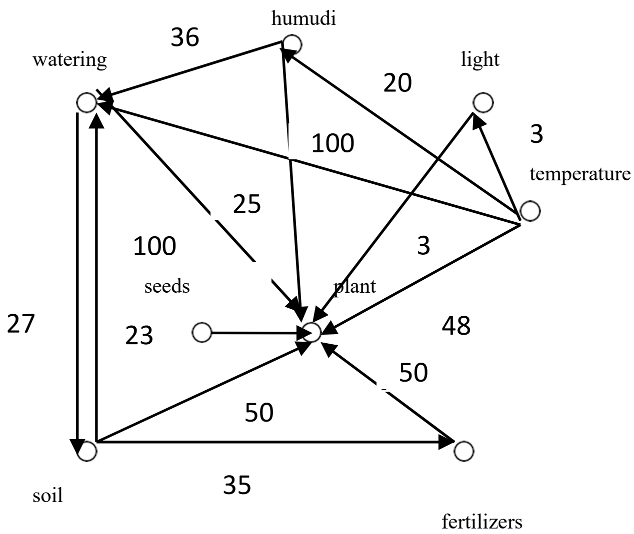
graph is called the effect graph of our agro-ecosystem [11]. We can extend our graphs, influence-diagrams to encode the whole agro-ecosystem into it, labelling the vertices by the quantities of the elements.



The effect graph can be analysed from different points of view, like temperature, light, humidity, watering, soil nature, seeds, fertilizer etc which effects the growth of the plant.In these investigations the most important (centre) element is the cultivated plant, and in the other vertices there are factors that effect the growth of the plant. We would like to examine how the agro-ecosystem takes up a new structure with the quantity change of some elements, turning our attention to the ‘centre’, the cultivated plant. With the help of the graph we can analyse how the quantities of other elements of the system change from time to time or how some elements compete for „food” with each other, as well.

Suppose the factors are given weigths according to their effects on the growth of the cultivated plant

Suppose one of the case such that the



The incidence matrix for the following effect graph is

plan	tempe	light	humud	watering	soil	fertilizer	seeds
48	0	3	20	100	0	0	0
3	0	0	0	0	0	0	0
100	0	0	0	36	0	0	0
25	0	0	0	0	27	0	0
50	0	0	0	23	0	35	0
50	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0

4. <u>WHAT HAPPENS IF ONE OF THE FACTORS IS ELIMINATED FROM THE SYSTEM?:</u>

What happens if one of the factors is eliminated from the system? (i.e. because of not watering the plant and climatically etc). In this case we assign a 0 weight to the appropriate edge connecting the central element (plant) . Unfortunately this is not enough. Indeed, the effects of other vertices can result nonzero entries to this element at later points of time, hence we have to change the weights of its edges (interactions) to 0. This will result a full 0 row and column in the matrix. Eliminating this row and columns we obtain a smaller matrix and we can use this for our model. Suppose the watering is made zero in the ecosystem then the growth of the plant will be effected

The corresponding incidence matrix becomes

plan	temp	light	hum	wate	soil	fertiliz	seeds
48	0	3	20	0	0	0	0
3	0	0	0	0	0	0	0
100	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
50	0	0	0	0	0	35	0
50	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0

5. <u>WHAT HAPPENS, IF SUDDENLY, UNEXPECTEDLY THE AMOUNT OF AN ELEMENT OF THE SYSTEM UNDESIREDLY INCREASES</u>

To eliminate the effect of the element which was undesirably increases such that to eliminate the effect of this item to our plant.. Then we have to examine, how we can alter the indirect interaction between these items. In our graph this can be implemented by examining all the paths from one vertex to the other and try to terminate the flow of effects on all these paths. In our graph this can be implemented by examining all the paths from one vertex to the other and try to terminate the flow of effects on all these paths. In graph theory language we have to disconnect the two vertices.

Suppose that the factor of temperature of the system increases then

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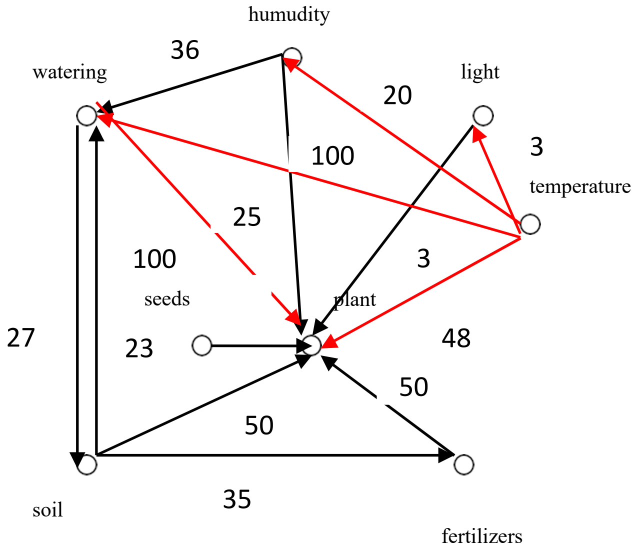


The factors which are effected by the sudden increase in the temperature are light, humidity, watering, growth of the plant. If we observe all the paths connected by the vertex temperature growth of the plant is possible only by the increase in the factor watering. Hence by examining all the paths of the graph we can decide what factors are to be added and deleted from the effect graph.

6. HOW TO FIND THE CHEAPEST AND FASTEST POSSIBLE WAY OF INCREASING THE YIELD IN THE REMAINING PERIOD OF TIME

This requires to find the fastest way to increase the yield with our leftover sources. In graph theory language it means finding the shortest paths with largest weights that can be done with e.g. Floyd's algorithm [12][13] Using Flyod's algorithm of graph theory which gives the shortest path with largest weights can be used for cheapest route for the cost of travelling graph and shortest route for distance graph. Here we modify the labelling. The weights will represent the quotient of the effect and the expenses of the desired interactions. The Floyd-Warshall algorithm is a popular algorithm for finding the shortest path for each vertex pair in a weighted directed graph. The Floyd–Warshall algorithm is an example of dynamic programming,The Floyd–Warshall algorithm compares all possible paths through the graph between each pair of vertices.

FLYOD'S ALGORITHM:

Construct a graph matrix for the given weighted graph where the order of the matrix is equal to the number of vertices

Let G = (V, E) with the vertices {1, 2, 3,..........., n} and the weight of the edge (i, j) is assigned a_{ij} in the graph matrix for the given weighted graph .The steps to find the shortest path from vertex 1 to the vertices 2, …., n is given as

Step 1: For each $i \in V(G)$

$a_{ii} = 0$, if there exist no loop for the vertices.

$a_{ii} = \infty$, If there is no edge connecting $i, j \in V(G)$

Step 2: For $k = 1\ to\ n$ do

For $i = 1\ to\ n$ do

For $j = 1\ to\ n$ do

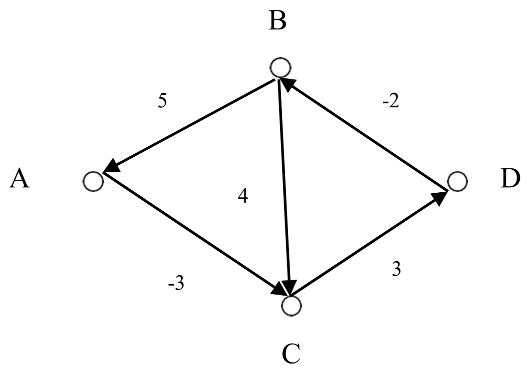
If $a_{ij} > a_{ik} + a_{kj}$ then

$a_{ij} \leftarrow a_{ik} + a_{kj}$

Else

$a_{ij} \leftarrow a_{ij}$

Step 3: The algorithm returns the shortest distance from each vertex to another in the given graph.Let us construct the shortest path for the given graph



Step 1 :

The graph matrix $A = \begin{bmatrix} 0 & \infty & -3 & \infty \\ 5 & 0 & 4 & \infty \\ \infty & \infty & 0 & 3 \\ \infty & -2 & \infty & 0 \end{bmatrix}$ Since $0\ k\ \infty + 5, 0\ k\ -3 + \infty, 0\ k\ \infty + \infty$

Therefore$a_{11} = 0$

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$$A = \begin{bmatrix} 0 & \infty & -3 & \infty \\ 5 & 0 & 4 & \infty \\ \infty & \infty & 0 & 3 \\ \infty & -2 & \infty & 0 \end{bmatrix} \quad since, \infty\ k\ \infty + 0\ but\ \infty > -3 + 0$$

Therefore $a_{12} = -3 + 0 = -3$

Therefore $$A = \begin{bmatrix} 0 & -3 & -3 & \infty \\ 5 & 0 & 4 & \infty \\ \infty & \infty & 0 & 3 \\ \infty & -2 & \infty & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & -3 & -3 & \infty \\ 5 & 0 & 4 & \infty \\ \infty & \infty & 0 & 3 \\ \infty & -2 & \infty & 0 \end{bmatrix} \quad \text{Since } -3\ k\ 0 + (-3), -3\ k\ (-3) + 4,$$

$$-3\ k\ (-3) + 4, -3\ k\ \infty +$$

Therefore $a_{13} = -3$

Therefore $$A = \begin{bmatrix} 0 & -3 & -3 & \infty \\ 5 & 0 & 4 & \infty \\ \infty & \infty & 0 & 3 \\ \infty & -2 & \infty & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & -3 & -3 & \infty \\ 5 & 0 & 4 & \infty \\ \infty & \infty & 0 & 3 \\ \infty & -2 & \infty & 0 \end{bmatrix} \quad since, \infty\ k\ \infty + 0, \infty\ k\ \infty + (-3)$$

$$but\ \infty > -3 + 3 = 0$$

Therefore $a_{14} = 0$

$$A = \begin{bmatrix} 0 & -3 & -3 & 0 \\ 5 & 0 & 4 & \infty \\ \infty & \infty & 0 & 3 \\ \infty & -2 & \infty & 0 \end{bmatrix}$$

Step 2:

Similarly computing we get $a_{21} = 5, a_{22} = -3, a_{23} = 4, a_{24} = 5$

$$A = \begin{bmatrix} 0 & -3 & -3 & 0 \\ 5 & -3 & 4 & 5 \\ \infty & \infty & 0 & 3 \\ \infty & -2 & \infty & 0 \end{bmatrix}$$

Step 3:

Similarly computing we get $a_{31} = \infty, a_{32} = 1, a_{33} = 0, a_{34} = 0$

$$A = \begin{bmatrix} 0 & -3 & -3 & 0 \\ 5 & -3 & 4 & 5 \\ \infty & 1 & 0 & 0 \\ \infty & -2 & \infty & 0 \end{bmatrix}$$

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Step 4:

Similarly computing we get $a_{41} = 3, a_{42} = -5, a_{43} = 2, a_{44} = 0$

$$A = \begin{bmatrix} 0 & -3 & -3 & 0 \\ 5 & -3 & 4 & 5 \\ \infty & 1 & 0 & 0 \\ 3 & -5 & 2 & 0 \end{bmatrix}$$

Step 5:

Similarly computing $a_{31} = 6$

Therefore

$$A = \begin{bmatrix} 0 & -3 & -3 & 0 \\ 5 & -3 & 4 & 5 \\ 6 & 1 & 0 & 0 \\ 3 & -5 & 2 & 0 \end{bmatrix}$$

Therefore the process is terminated as all the elements are not infinity

Hence we find the shortest distance from all the vertices of the graph.

Example 2 : To find the shortest path from Camden to all the cities Trenton, Woodbridge, Asbury Park, Atlantic City, Capemay in Figure 2.

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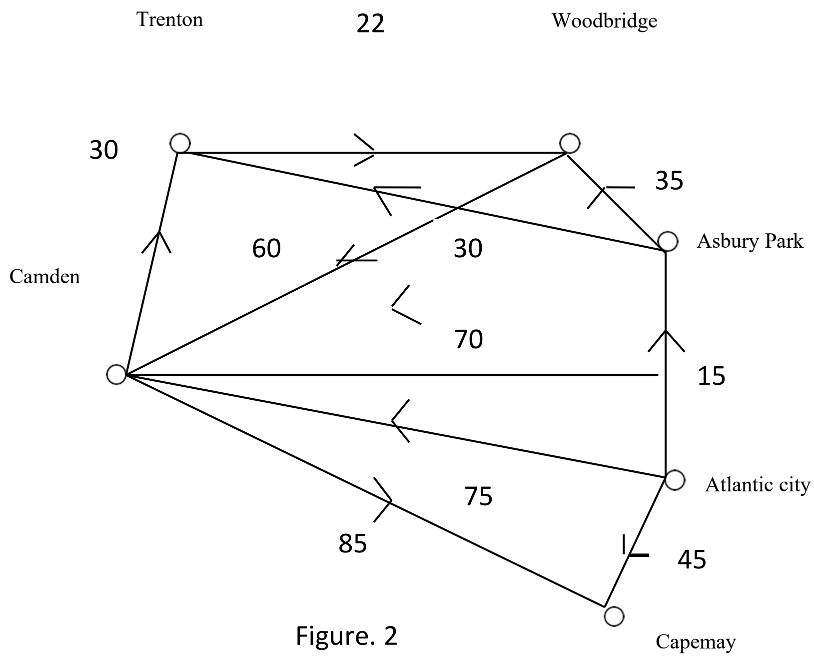

Figure. 2

The Graph matrix for the above graph is

$$\begin{bmatrix} 0 & 30 & \infty & \infty & \infty & 85 \\ \infty & 0 & 22 & \infty & \infty & \infty \\ 60 & \infty & 0 & \infty & \infty & \infty \\ 70 & \infty & 35 & 0 & \infty & \infty \\ 75 & \infty & \infty & 15 & 0 & 45 \\ \infty & \infty & \infty & \infty & \infty & 0 \end{bmatrix}$$

Step 1 : After the first iteration the matrix becomes

$$\begin{bmatrix} 0 & 0 & 22 & \infty & 85 & 85 \\ 82 & 0 & 0 & 22 & \infty & \infty \\ \infty & 90 & \infty & 0 & \infty & 145 \\ 60 & 100 & 0 & 0 & 0 & 155 \\ 70 & 105 & 15 & 0 & 0 & 0 \\ 75 & \infty & \infty & 0 & 0 & 0 \end{bmatrix}$$

Step 2 : Applying the procedure since there are elements containing ∞

$$\begin{bmatrix} 0 & 0 & 22 & 22 & 85 & 85 \\ 82 & 0 & 0 & 22 & 22 & 167 \\ 60 & 90 & 0 & 0 & \infty & 145 \\ 60 & 75 & 0 & 0 & 0 & 0 \\ 60 & 75 & 0 & 0 & 0 & 0 \\ 60 & 75 & 15 & 0 & 0 & 0 \end{bmatrix}$$

Which gives the shortest path between the cities

CONCLUSION:

When extending the model, it integrates biological, technological and economical factors and joins the natural circumstances. Agricultural production meeting several requirements su became more and more in focus. Nowadays investigation of influences explained by climate change and climate variability is of interest and needed. If we are interested in more precise details, the elements of our graph can be considered as subgraphs of the effectgraph with the similar structure. Our center element, the cultivated plant as a subgraph can consist of the elements representing the relative water holding capacity, the evaporation transpiration of the plant, the biomass growth, etc. The soil element as a subgraph can consist of e.g. water content of soil, different nutrient content of it, temperature of different layers of the soil, the water run-off in it, the evaporation or the water holding capacity of soil. In the same way variables as temperature, percipitation and others can also be included. This way we can get a very complex structure of the agro-ecosystem. The Effect graph can be constructed in

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different models of the ecosystem. There are several different models to find the shortest path in graph theory.

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