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Propagation Of Shock Waves Through A Channel Of Varying Area Of Cross- Section Containing Mixture Of A Gas And Small Solid Particles

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Abstract--the problem of propagation of shock waves has been discussed in a gas containing small solid particles through a tube of variable area of cross-section by using the method developed by Whitham. The mixture of gas and solid particles is assumed to be in viscid and the flow is governed by Euler's equations expressing conservation of mass, momentum and energy. The results derived here are generalization of the results obtained previously in absence of small solid particles. Effects of a change in the value of parameters of small solid particles kp and G on the shock propagation are discussed.

Keywords— Small solid particle (assumed in viscid), Euler' s equation, Spherical and cylindrical Shock waves.

I. INTRODUCTION

The study of the shock wave propagation through a gaseous medium has a wide range of applications in engineering problems, applied physics and in fluid dynamics. The problem of converging shock waves and detonation waves are the promising means to produce an extreme condition of very high temperature, pressure and density. This problem of converging shock waves was first presented and discussed by Guderley [1]. \

This study gives a shock of infinite strength at the center of convergence. A numerical solution for a converging cylindrical shock was given by Payne [2]. The problem of a contracting spherical or cylindrical shock front propagation in to a uniform gas at rest was studied by Stanyukovich [3]. The problems of implosion of a spherical shock wave in a gas and the collapse of a spherical bubble in a liquid are discussed by Zeldovich and Raizer [4] by using self-similar solution method. Matsuo [5] has described the whole history of the fluid motion from the initial stage to the focusing stage by using non-similar approximated method.

Whitham [6] has given a very simple and effective rule for the analysis of imploding shocks. A large number of problems have been discussed with the help of this rule, for example, the description of propagation process of shock waves, through a channel of variable section (Chisnell [7]), in a medium of variable density (Tyl and Wlodarczyk [8]), in a non-ideal gas (Ojha and Tewari [9]), in astrophysical atmosphere (Bird [10], Ojha and Singh [11], Ojha and Nath [12], Ojha and Singh [13], Ojha and Tewari [14]) and in the analysis of imploding detonation waves (Lee and Lee [15], Singh [16]). Although the Whitham rule is approximated but it agrees well with exact solutions and with experimental results (Lee [17], Guy [18], Jumper [19]).

In many astrophysical situations and in engineering problems, it is necessary to consider the high speed of the flow of a mixture of gas and small solid particles (Miura and Glass [20], Pai et al. [21], Vishwakarma and pandey [22]). Our aim here is to discuss the propagation of shock waves through a tube of variable area of cross-section containing a mixture of a gas and small solid particles. The mixture model used here is similar to that of Pai et al [21], Steiner and Hirschler [23].

In course of discussion, we have used the method developed by Whitham [6]. The fundamental equations and the shock conditions across the shock are summarized here in section 2 from a number of investigations such as; Pai et al [21], Steiner and Hirschler [23], Vishwakarma and Pandey [22]. Discussion of the problem are given in section 3 and the results are in section 4.

II. BASIC EQUATIONS

The fundamental equations for the one dimensional unsteady flow of a mixture of a gas and small solid particles flowing in a channel of variable area of cross-section A can be written as

(2.1)
$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + \rho\left(\frac{\partial u}{\partial x} + \frac{u}{A}\frac{\partial A}{\partial x}\right) = 0 ,$$

(2.2)
$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right) = -\frac{\partial p}{\partial x} ,$$

(2.3)
$$\frac{\partial e}{\partial t} + u\frac{\partial e}{\partial x} - \frac{p}{\rho^2}\left(\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x}\right) = 0 ,$$

where ρ is density of the mixture, u the flow velocity, p the pressure, e the internal energy per unit mass of the mixture, x the distance along the channel and t the time.

Let us assume the flow to be a gas mixture obeying the equation of state of Mie-Grüneisen type (Pai et al. [21]), (2.4) $$p = \frac{(1-k_p)\rho R^* T}{1-z},$$

Where R* be the gas constant, z the volume fraction of solid particles in the mixture and kp is the mass concentration of the solid particles. The relation between kp and z is given by,

(2.5) $$k_p = \frac{z}{\rho}\rho_{sp}$$ and

(2.6) $$z_0 = \frac{k_p}{G(1-k_p)+k_p}, \quad \frac{z}{\rho} = \frac{z_0}{\rho_0},$$

where ρsp is the species density of the solid particles, z0 and ρ0 are the initial values of z and ρ respectively, and G the ratio of the density of solid particles to the initial density of gas.

The internal energy e of the mixture may be written as

(2.7) e = [kp Csp + (1– kp) Cv]T = Cvm T,

where Csp is the specific heat of solid particles, Cv the specific heat of the gas at constant volume and Cvm the specific heat of the mixture at constant volume.

The specific heat of the mixture at constant pressure is

(2.8) Cpm = kp Csp + (1– kp) Cp,

where Cp is the specific heat of the gas at constant pressure.

The ratio of the specific heat of the mixture is given by

(2.9) $$\Gamma = \frac{C_{pm}}{C_{vm}} = \frac{\gamma+\delta\beta}{1+\delta\beta},$$

where $\gamma = \frac{C_p}{C_v}$, $\delta = \frac{k_p}{1-k_p}$ and $\beta = \frac{C_{sp}}{C_v}$.

The internal energy e of the mixture is, therefore, given by

(2.10) $$e = \frac{p(1-z)}{\rho(\Gamma-1)}.$$

The speed of sound a, in the mixture is defined by

(2.11) $$a^2 = \frac{\Gamma p}{\rho(1-z)}.$$

With the help of (2.10), (2.11) and (2.1) equation (2.3) may be written as

(2.12)
$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + a^2\left(\frac{\partial u}{\partial x} + \frac{u}{A}\frac{\partial A}{\partial x}\right) = 0.$$

Let a strong shock wave be propagating into the homogeneous gas mixture of constant density ρ_0. Again let the gas mixture be at rest with negligibly small counter pressure. Then, the relations between the quantities in front and behind the shock in a homogeneous mixture are (Steiner and Hirshchler [23], Vishwakarma and Pandey [22]),

(2.13) $$\rho_0 U = \rho_1 (U - u_1),$$

(2.14) $$\rho_0 U^2 = p_1 + \rho_1 (U - u_1)^2,$$

(2.15) $$(1/2)U^2 = e_1 + \frac{p_1}{\rho_1} + (1/2)(U - u_1)^2.$$

Equations (2.13) and (2.14) can be written as,

(2.16) $$\rho_1 = \frac{\rho_0}{\left(1-\frac{u_1}{U}\right)}$$

and

(2.17) $$\frac{p_1}{\rho_0 U^2} = \frac{u_1}{U}.$$

Introducing (2.10), (2.16) and (2.17) into (2.15), yields after some manipulation

(2.18) $$u_1 = \frac{2(1-z_0)U}{(\Gamma+1)}$$

(2.19) $$\rho_1 = \frac{\rho_0(\Gamma+1)}{(\Gamma-1+2z_0)}$$

(2.20) $$p_1 = \left(\frac{2}{\Gamma+1}\right)\rho_0(1-z_0)U^2.$$

III. DISCUSSION OF THE PROBLEM

Now we apply the Whitham's rule to determine the shock wave propagation through a channel of variable section containing a mixture of gas and small solid particles. By using equations (2.1), (2.2) and (2.12) we may write,

(3.1)

$$\frac{\partial p}{\partial t}+(u\pm a)\frac{\partial p}{\partial x}\pm\rho a\left(\frac{\partial u}{\partial t}+(u\pm a)\frac{\partial u}{\partial x}\right)+\frac{\rho a^2 u}{A}\frac{\partial A}{\partial x}=0$$

In characteristic form (4.1) may be written as

(3.2) $$dp + \rho\, a\, du + \frac{\rho a^2 u}{u+a}\frac{dA}{A} = 0 \quad \text{along} \quad \frac{dx}{dt} = u + a$$

and

(3.3) $$dp - \rho\, a\, du + \frac{\rho a^2 u}{u-a}\frac{dA}{A} = 0 \quad \text{along} \quad \frac{dx}{dt} = u - a.$$

The Whitham's rule states that when the relevant equations are written first in the characteristic form, the differential relation which must be satisfied along a characteristic can be applied to the flow quantities just behind the shock wave. Together with the shock relations, this rule determines the motion of the shock wave. We assume here that the shock relations to hold, of course, within the order of approximation determined by a constant value of U. We apply here the differential relation (3.3) along the characteristic behind the shock wave. Together with the shock conditions we are able to describe U or related quantities in terms of equilibrium quantities. Using (3.11), (3.18), (3.19) and (3.20) we find,

(3.4)

$$\rho_1 a_1=\rho_0\left\{\frac{2\Gamma(1-z_0)}{(\Gamma-1+2z_0)-z_0(\Gamma+1)}\right\}^{1/2}$$

and

(3.5)

$$\frac{\rho_1 u_1 a_1{}^2}{u_1-a_1}=\frac{\dfrac{4\Gamma\rho_0 U^2(1-z_0)^2(\Gamma-1+2z_0)}{(\Gamma+1)^2\{\Gamma-1+2z_0)-z_0(\Gamma+1)\}}}{\dfrac{2(1-z_0)}{(\Gamma+1)}-\left(\dfrac{\Gamma-1+2z_0}{\Gamma+1}\right)\sqrt{\dfrac{2\Gamma(1-z_0)}{(\Gamma-1+2z_0)-z_0(\Gamma+1)}}}$$

Substitution of (4.4) and (4.5) into (4.3) gives

(3.6) $$\frac{dU}{U}=-K(z_0)\frac{dA}{A},$$

where

(3.7) $K(z_0)$ =

$$\left[2\Gamma(1-z_0)(\Gamma-1+2z_0)\right.$$

$$\left\{\sqrt{(\Gamma-1+2z_0)-z_0(\Gamma+1)}\right\}^{-1}\times$$

$$\left\{2(1-z_0)\sqrt{(\Gamma-1+2z_0)-z_0(\Gamma+1)}-(\Gamma\right.$$

.

The law of propagation of shock wave can be determined by integration of (4.6). For a certain value of $K(z_0)$ the integration of (4.6) gives

(3.8) $$\frac{U}{\overline{U}}=\left(\frac{A}{\overline{A}}\right)^{-K(z_0)},$$

where $\overline{U}$ and $\overline{A}$ are the values of U and A at some reference position.

The corresponding laws for the variation of u_1 and p_1 are given by

(3.9) $$\frac{u_1}{\overline{U}}=\frac{2(1-z_0)}{(\Gamma+1)}\left(\frac{A}{\overline{A}}\right)^{-K(z_0)}$$

and

(3.10)

$$\frac{p_1}{\rho_0\overline{U}^2}=\frac{2(1-z_0)}{(\Gamma+1)}\left(\frac{A}{\overline{A}}\right)^{-2K(z_0)}.$$

In absence of solid particles, the value of $K(z_0)$ takes the form $K(\gamma)$ as,

(3.11)

$$K(\gamma)=\frac{\gamma}{\left(1-\sqrt{\dfrac{\gamma(\gamma-1)}{2}}\right)\left(1-\sqrt{\dfrac{2\gamma}{\gamma-1}}\right)}$$

which is equivalent to the expression obtained in case of ideal gas.

Thus, effects of presence of small solid particles on the propagation of shock waves and on the flow variables behind the shock waves can be obtained from the expressions (4.8), (4.9) and (4.10).

IV. RESULTS

Variation of the non-dimensional shock velocity $\frac{U}{\overline{U}}$, the non-dimensional fluid velocity $\frac{u_1}{\overline{U}}$ and the non-dimensional pressure $\dfrac{p_1}{\rho_0 \overline{U}^2}$ with the non-dimensional channel cross-section $\dfrac{A}{\overline{A}}$ are obtained from equations (4.8), (4.9) and (4.10) and plotted in figures 1 to 3 for dust-free case and for G = 10and 100, and in figures 4 to 6 for G = 1. Values of the constant parameters are taken to be $\gamma = 1.4$; $\beta = 1$; $k_p = 0, 0.2, 0.4$; and $G = 1, 10, 100$. The value $k_p = 0$ corresponds to the case of dust-free perfect gas. The value G = 1 corresponds to $z_0 = K_p$, i.e. the case when initial volume fraction of solid particles in the mixture is equal to the mass fraction of solid particles.

Figures 1-3 show that the shock velocity $\dfrac{U}{\overline{U}}$, fluid velocity $\dfrac{u_1}{\overline{U}}$ and pressure $\dfrac{p_1}{\rho_0 \overline{U}^2}$ remain almost constant during some distance, and then start to increase rapidly and tends to infinity as the point of convergence is approached.

On the other hand, figures 4-6 show that, as $\dfrac{A}{\overline{A}}$ decreases, these variables decrease and tend to zero as the point of convergence is approached. Thus the behaviour of the variables $\dfrac{U}{\overline{U}}$, $\dfrac{u_1}{\overline{U}}$, $\dfrac{p_1}{\rho_0 \overline{U}^2}$ for G = 1 are in contrast with those for dust-free case and for G = 10, 100. This phenomenal behaviour of variables may be, physically, interpreted as follows.

In fact, at G = 1, the initial volume fraction of solid particles in the mixture z_0 is equal to k_p, as mentioned earlier, and therefore at higher values of k_p, z_0 is higher. This means a significant portion of mixture (20 % or 40% of the total volume, when G = 1 and k_p = 0.2 or 0.4) is occupied by small solid particles causing a heavy loss in compressibility of the medium, which is responsible for strong decay in shock velocity and other variables.

Figures 1-3 show that the effects of an increase in k_p depends on the value of G. When G = 10, an increase in k_p results in an early growth of the flow variables, whereas when G = 100, it results in somewhat delayed growth of the variables.

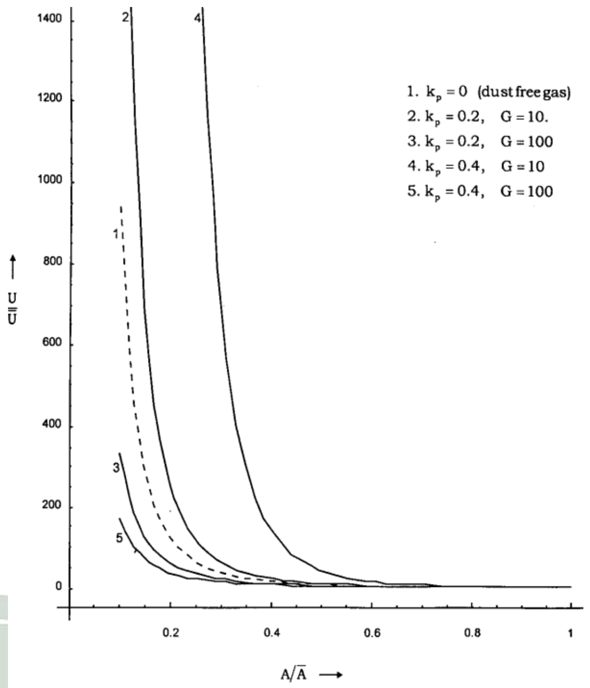

Fig. 1. Variation of shock velocity with channel cross-section for different values of k_p and G

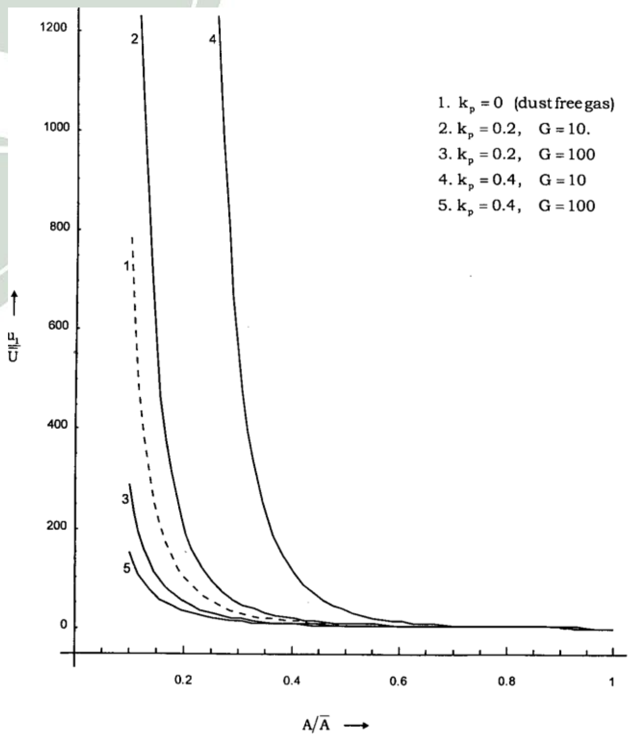

Fig. 2. Variation of fluid velocity just behind the shock with channel cross-section for different values of k_p and G

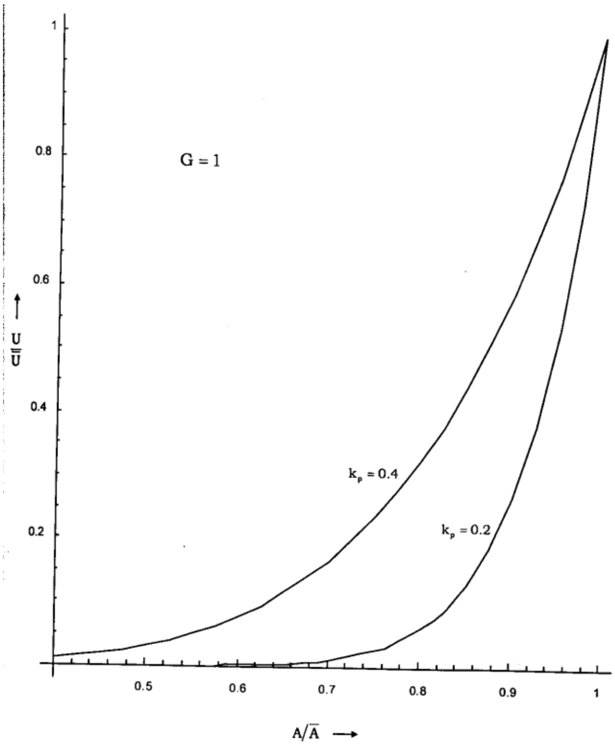

Fig. 4. Variation of shock velocity with channel cross-section for G = 1

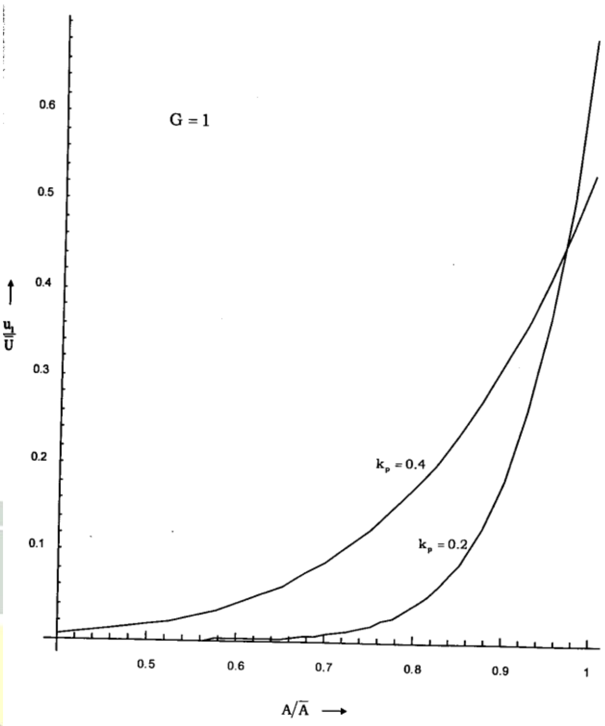

Fig. 5. Variation of fluid velocity just behind the shock with channel cross-section for G = 1

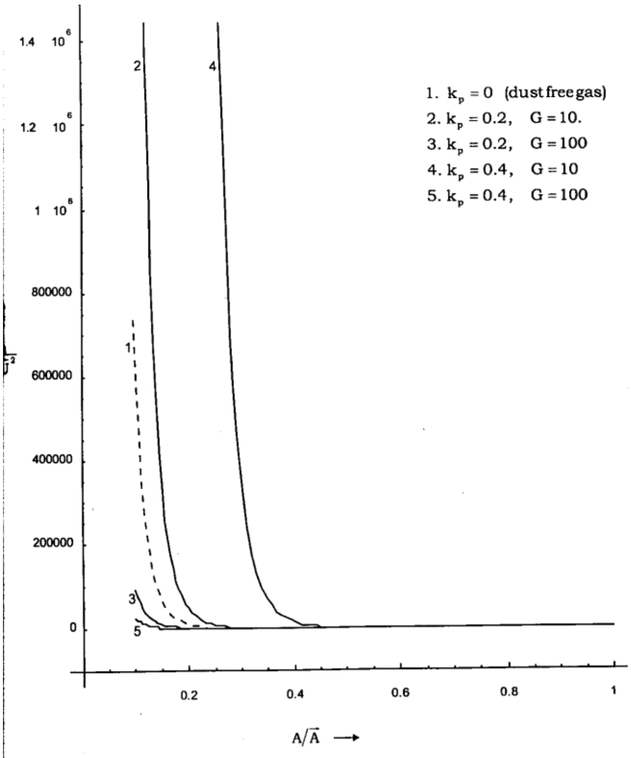

Fig. 3. Variation of pressure just behind the shock with channel cross-section for different values of k_p and G

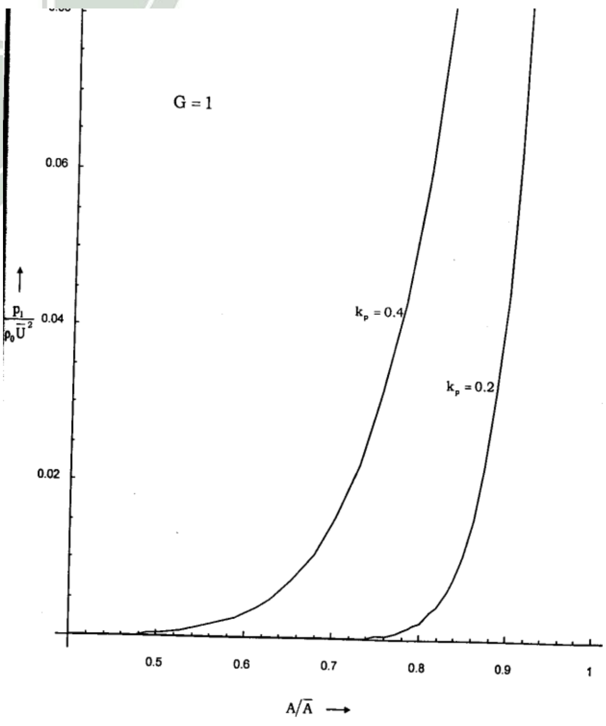

Fig. 6. Variation of pressure just behind the shock with channel cross-section for G = 1

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