

Contra fuzzy lattice KS Operator group

Mr. M. U. Makandar

Assistant Professor

PG, MBA Department, KIT's IMER, Kolhapur. Maharashtra

Abstract:

In this paper a fuzzy set is defined on a group with two operators which is also a lattice satisfying four conditions. The operator sets are denoted by K and S which are any nonempty sets. The first two conditions are according to group structure and the last two conditions are according to lattice structure.

Keywords: Lattice group, Contra Fuzzy lattice group, Contra Fuzzy lattice KS operator group.

Introduction

A fuzzy algebra has become an important branch of research. A. Rosenfeld 1971 [9] used the concept of fuzzy set theory due to Zadeh 1965 [5]. Since then the study of fuzzy algebraic substructures are important when viewed from a Lattice theoretic point of view. N. Ajmal and K.V. Thomas [1] initiated such types of study in the year 1994. It was latter independently established by N. Ajmal [1] that the set of all fuzzy normal subgroups of a group constitute a sub lattice of the lattice of all fuzzy sub groups of a given group and is Modular. Nanda[8] proposed the notion of fuzzy lattice using the concept of fuzzy partial ordering. More recently in the notion of set product is discussed in details and in the lattice theoretical aspects of fuzzy sub groups and fuzzy normal sub groups are explored. G.S.V. Satya Saibaba [3] initiate the study of L-fuzzy lattice ordered groups and introducing the notice of L-fuzzy sub l- groups. J.A. Goguen [4] replaced the valuation set [0,1] by means of a complete lattice in an attempt to make a generalized study of fuzzy set theory by studying L-fuzzy sets. A Solairaju and R. Nagarajan [11] introduced the concept of lattice valued Q-fuzzy sub-modules over near rings with respect to T-norms. DrM.Marudai & V. Rajendran[6] modified the definition of fuzzy lattice and introduce the notion of fuzzy lattice of groups and investigated some of its basic properties. Gu [12] introduced concept of fuzzy groups with operator. Then S. Subramanian, R Nagarajan & Chellappa [10] extended the concept to m fuzzy groups with operator. In this paper we introduce the notion of Contra fuzzy lattice KS operator group and some of its properties.

1. PRELIMINARIES

Definition 1.1 Contra Fuzzy group

Let $\lambda: X$ to $[0, 1]$ is a fuzzy set & (G, \cdot) is a group which is a subset of X . Fuzzy group is a fuzzy set which satisfy two conditions

- 1) $\lambda(x \cdot y) \leq \max\{\lambda(x), \lambda(y)\}$
- 2) $\lambda(x^{-1}) \leq \lambda(x)$ where $x, y \in G$.

Definition 1.2 K-Operator group

A group G is said to be a K- operator group if $kx \in G$ where $k \in K$ (any non empty set called as Operator set) and for all $x \in G$.

Definition 1.3 Contra Fuzzy K- operator group

Let $\lambda: X$ to $[0, 1]$ is a fuzzy set & G is a subset of X which is also a K - operator group. λ is a fuzzy K - operator group if it satisfy following two conditions

- i) $\lambda(k(xy)) \leq \max\{\lambda(kx), \lambda(ky)\}$
- ii) $\lambda(kx)^{-1} \leq \lambda(kx)$ where $x, y \in G, k \in K$.

Definition 1.4 Lattice K-operator group

Lattice K -operator group is an algebraic structure (G, \cdot, R) if it satisfy two conditions 1) G is K -operator group w.r.t ' \cdot ' 2) G is a lattice w.r.t R

Definition 1.5 Contra Fuzzy lattice K-operator group

$\lambda: X$ to $[0, 1]$ is a fuzzy set; G is a K - lattice operator group, A function λ on G is said to be a Contra fuzzy lattice K -operator group if it satisfy following four conditions

- 1) $\lambda(kxky) \leq \max\{\lambda(kx), \lambda(ky)\}$
- 2) $\lambda(kx)^{-1} \leq \lambda(kx)$
- 3) $\lambda(kx \vee ky) \leq \max\{\lambda(kx), \lambda(ky)\}$
- 4) $\lambda(kx \wedge ky) \leq \max\{\lambda(kx), \lambda(ky)\}$ For all $x, y \in G, k \in K$

Definition 1.6 KS- operator group

Let G be a group, K, S be any two nonempty sets if $kx \in G, sx \in G$. for every $x \in G, k \in K, s \in S$ Then G is called KS - operator group.

Definition 1.7 Contra Fuzzy KS- operator group

If $\lambda: X$ to $[0, 1]$ is a fuzzy set & G is KS - operator group . A fuzzy set λ over G, G subset of X is a Contra fuzzy KS operator group if

- 1) $\lambda(kxsy) \leq \max\{\lambda(kx), \lambda(sy)\}$ 2) $\lambda(kx)^{-1} \leq \lambda(kx)$ & $\lambda(sx)^{-1} \leq \lambda(sx)$ for every $x, y \in G, k \in K, s \in S$

Definition 1.8 Lattice KS operator group

A lattice KS - operator group is an algebraic structure (G, R, \cdot) if it satisfy two conditions 1) G is a KS - operator group w.r.t ' \cdot ' 2) G is a lattice w.r.t R .

Definition 1.9 Contra Fuzzy lattice KS- operator group (CFL KS- operator group)

$\lambda: X$ to $[0, 1]$ be a fuzzy set, Let G be a subset of X which is a lattice KS - operator group , K, S (operator sets). λ is a function over G . It is a Contra fuzzy lattice KS - operator group if it satisfy following four conditions

- 1) $\lambda(kxsy) \leq \max\{\lambda(kx), \lambda(sy)\}$
- 2) $\lambda(kx)^{-1} \leq \lambda(kx)$ & $\lambda(sx)^{-1} \leq \lambda(sx)$
- 3) $\lambda(kx \vee sy) \leq \max\{\lambda(kx), \lambda(sy)\}$
- 4) $\lambda(kx \wedge sy) \leq \max\{\lambda(kx), \lambda(sy)\}$ For every $x \in G, k \in K, s \in S$

Definition 1.10 Contra Fuzzy lattice KK -operator group

$\lambda: X$ to $[0, 1]$ is a fuzzy set; G is a K - lattice operator group, A function λ on G is said to be a Contra fuzzy lattice KK -operator group if it satisfy following four conditions

- 1) $\lambda(k_1x k_2y) \leq \max\{\lambda(k_1x), \lambda(k_2y)\}$
- 2) $\lambda(k_1x)^{-1} \leq \lambda(k_1x), \lambda(k_2x)^{-1} \geq \lambda(k_2x),$
- 3) $\lambda(k_1x \vee k_2y) \leq \max\{\lambda(k_1x), \lambda(k_2y)\}$

$$4) \quad \lambda(k_1x \wedge k_2y) \leq \max\{\lambda(k_1x), \lambda(k_2y)\}, \text{ For all } x, y \in G, k_1, k_2 \in K$$

Definition 1.11 **Contra Fuzzy lattice K²-operator group**

$\lambda: X$ to $[0, 1]$ is a fuzzy set; G is a K - lattice operator group, A function λ on G is said to be a Contra fuzzy lattice K²-operator group if it satisfy following four conditions

- 1) $\lambda(kxky) \leq \max\{\lambda(kx), \lambda(ky)\}$
- 2) $\lambda(kx)^{-1} \leq \lambda(kx)$
- 3) $\lambda(kx \vee ky) \leq \max\{\lambda(kx), \lambda(ky)\}$
- 4) $\lambda(kx \wedge ky) \leq \max\{\lambda(kx), \lambda(ky)\}$ For all $x, y \in G, k \in K$

Definition 1.12

Let $\lambda: X$ to Y be a function. Q is a fuzzy group of Y . A fuzzy set λ^{-1} Inverse image of Q under λ is given by $\lambda^{-1}(Q) = \mu_{\lambda^{-1}(Q)}(x) = \mu_Q(\lambda(x))$

Definition 1.13

$\mu_A: X$ to $[0, 1]$ be a fuzzy set and $\lambda: X \rightarrow X'$ is a function. A function $\mu_{A\lambda}: X$ to $[0,1]$ is defined by $\mu_{A\lambda}(x) = \mu_A(\lambda(x))$

Definition 1.14 If T and T' are lattice KS- operator groups .A function

$\lambda: T$ to T' be a lattice KS homomorphism

if $\lambda(kxsy) = \lambda(kx)\lambda(sy) = k\lambda(x)s\lambda(y), \lambda(kx \vee sy) = \lambda(kx) \vee \lambda(sy) = k\lambda(x) \vee s\lambda(y), \lambda(kx \wedge sy) = \lambda(kx) \wedge \lambda(sy) = k\lambda(x) \wedge s\lambda(y)$ For all $x, y \in G, k \in K, s \in S$

Definition 1.15

Let A_i be a contra fuzzy lattice KS operator groups of G_i , for $i = 1, 2, \dots, n$. Then the product A_i ($i = 1, 2, \dots, n$) is the function $A_1 \times A_2 \times \dots \times A_n: G_1 \times G_2 \times \dots \times G_n \rightarrow [0,1]$ defined by

$$(A_1 \times A_2 \times \dots \times A_n)(k(x_1, x_2, \dots, x_n)) = \max\{A_1(kx_1), A_2(kx_2), \dots, A_n(kx_n)\}$$

2. PROPERTIES OF CONTRA FL KS- OPERATOR GROUP

Proposition 2.1: Let T and T' be two Lattice KS-operator groups and $\lambda: T$ to T' be a lattice KS homomorphism. If P' is a Contra FL KS operator group of T' then the pre-image $\lambda^{-1}(P')$ is also a Contra FL KS operator group of T .

Proof- Assume P' is a Contra FL KS- operator group of T' . Let $x, y \in T$

- i) $\lambda^{-1}(P') = \mu_{\lambda^{-1}(P')}(kxsy) = \mu_{P'}(\lambda(kxsy)) = \mu_{P'}(k\lambda(x)s\lambda(y)) \leq \max\{\mu_{P'}(k\lambda(x)), \mu_{P'}(s\lambda(y))\} \leq \max\{\mu_{P'}(\lambda(kx)), \mu_{P'}(\lambda(sy))\} = \max\{\mu_{\lambda^{-1}(P')}(kx), \mu_{\lambda^{-1}(P')}(sy)\}$
- ii) $\mu_{\lambda^{-1}(P')}(kx)^{-1} = \mu_{P'}\lambda(kx)^{-1} = \mu_{P'}[\lambda(kx)]^{-1} = \mu_{P'}[k\lambda(x)]^{-1} \leq \mu_{P'}k\lambda(x) = \mu_{P'}(\lambda(kx)) = \mu_{\lambda^{-1}(P')}(kx)$
 $\mu_{\lambda^{-1}(P')}(sx)^{-1} = \mu_{P'}\lambda(sx)^{-1} = \mu_{P'}[\lambda(sx)]^{-1} = \mu_{P'}[s\lambda(x)]^{-1} \leq \mu_{P'}s\lambda(x) = \mu_{P'}(\lambda(sx)) = \mu_{\lambda^{-1}(P')}(sx)$
- iii) $\mu_{\lambda^{-1}(P')}(kx \vee sy) = \mu_{P'}\lambda(kx \vee sy) = \mu_{P'}\lambda(kx) \vee \lambda(sy) \leq \max\{\mu_{P'}\lambda(kx), \mu_{P'}\lambda(sy)\} \leq \max\{\mu_{\lambda^{-1}(P')}(kx), \mu_{\lambda^{-1}(P')}(sy)\}$
- iv) $\mu_{\lambda^{-1}(P')}(kx \wedge sy) = \mu_{P'}\lambda(kx \wedge sy) = \mu_{P'}\lambda(kx) \wedge \lambda(sy) \leq \max\{\mu_{P'}\lambda(kx), \mu_{P'}\lambda(sy)\}$

$$\leq \max_i \{ \mu_{\lambda^{-1}(P')}(kx), \mu_{\lambda^{-1}(P')}(sy) \}$$

Therefore $\lambda^{-1}(P')$ is a Contra FL KS- operator group of T.

Proposition 2.2: If T and T' are two Lattice KS operator groups and $\lambda: T$ to T' is a lattice KS epimorphism. P' is a fuzzy set in T'. If $\lambda^{-1}(P')$ is a Contra FL KS operators group of T then P' is a Contra FL KS- operators group of T'.

Proof- Consider x, y $\in T'$, hence there is are elements m, n $\in T$ so that $\lambda(m) = x$ and $\lambda(n) = y$.

$$\begin{aligned} \text{i)} \quad & \mu_{P'}(kx sy) = \mu_{P'}(k\lambda(m) s\lambda(n)) = \mu_{P'}(\lambda(kmsn)) = \mu_{\lambda^{-1}(P')}(kmsn) \\ & \leq \max_i \{ \mu_{\lambda^{-1}(P')}(km), \mu_{\lambda^{-1}(P')}(sn) \} \leq \max_i \{ \mu_{P'}\lambda(km), \mu_{P'}\lambda(sn) \} \\ & \leq \max_i \{ \mu_{P'}k\lambda(m), \mu_{P'}s\lambda(n) \} \leq \max_i \{ \mu_{P'}(kx), \mu_{P'}(sy) \} \\ \text{ii)} \quad & \mu_{P'}(kx)^{-1} = \mu_{P'}(k\lambda(m))^{-1} = \mu_{P'}(\lambda(km))^{-1} = \mu_{\lambda^{-1}(P')}(km)^{-1} \leq \mu_{\lambda^{-1}(P')}(km) \\ & = \mu_{P'}\lambda(km) = \mu_{P'}k\lambda(m) = \mu_{P'}(kx) \\ & \mu_{P'}(sx)^{-1} = \mu_{P'}(s\lambda(m))^{-1} = \mu_{P'}(\lambda(sm))^{-1} = \mu_{\lambda^{-1}(P')}(sm)^{-1} \leq \mu_{\lambda^{-1}(P')}(sm) \\ & = \mu_{P'}\lambda(sm) = \mu_{P'}s\lambda(m) = \mu_{P'}(sx) \\ \text{iii)} \quad & \mu_{P'}(kx \vee sy) = \mu_{P'}(k\lambda(m) \vee s\lambda(n)) = \mu_{P'}(\lambda(km) \vee \lambda(sn)) = \mu_{P'}(\lambda(km \vee sn)) \\ & = \mu_{\lambda^{-1}(P')}(km \vee sn) \leq \max_i \{ \mu_{\lambda^{-1}(P')}(km), \mu_{\lambda^{-1}(P')}(sn) \} \\ & = \max_i \{ \mu_{P'}\lambda(km), \mu_{P'}\lambda(sn) \} = \max_i \{ \mu_{P'}k\lambda(m), \mu_{P'}s\lambda(n) \} \\ & = \max_i \{ \mu_{P'}(kx), \mu_{P'}(sy) \} \\ \text{iv)} \quad & \mu_{P'}(kx \wedge sy) = \mu_{P'}(k\lambda(m) \wedge s\lambda(n)) = \mu_{P'}(\lambda(km) \wedge \lambda(sn)) = \mu_{P'}(\lambda(km \wedge sn)) \\ & = \mu_{\lambda^{-1}(P')}(km \wedge sn) \leq \max_i \{ \mu_{\lambda^{-1}(P')}(km), \mu_{\lambda^{-1}(P')}(sn) \} \\ & = \max_i \{ \mu_{P'}\lambda(km), \mu_{P'}\lambda(sn) \} = \max_i \{ \mu_{P'}k\lambda(m), \mu_{P'}s\lambda(n) \} \\ & = \max_i \{ \mu_{P'}(kx), \mu_{P'}(sy) \} \end{aligned}$$

Therefore P' is a Contra FL KS operator group of T'.

Proposition 2.3: If $\{A_i\}$ is a family of Contra FL KS operator group of T then $\bigcup A_i$ is also a Contra FL KS operator group of T where $\bigcup A_i = \{ x, \vee \lambda_{A_i}(x) / x \in T \}$

Proof- Consider x, y $\in T$

$$\begin{aligned} \text{i)} \quad & (\bigcup \lambda_{A_i})(kx sy) = \vee \lambda_{A_i}(kx sy) \leq \vee \max_i \{ \lambda_{A_i}(kx), \lambda_{A_i}(sy) \} \\ & = \max_i \{ (\bigcup \lambda_{A_i})(kx), (\bigcup \lambda_{A_i})(sy) \} \\ \text{ii)} \quad & (\bigcup \lambda_{A_i})(kx^{-1}) = \vee \lambda_{A_i}(kx)^{-1} \leq \vee \lambda_{A_i}(kx) = (\bigcup \lambda_{A_i})(kx) \\ & (\bigcup \lambda_{A_i})(sx^{-1}) = \vee \lambda_{A_i}(sx)^{-1} \leq \vee \lambda_{A_i}(sx) = (\bigcup \lambda_{A_i})(sx) \\ \text{iii)} \quad & (\bigcup \lambda_{A_i})(kx \vee sy) = \vee \lambda_{A_i}(kx \vee sy) \leq \vee \max_i \{ \lambda_{A_i}(kx), \lambda_{A_i}(sy) \} \\ & = \max_i \{ (\bigcup \lambda_{A_i})(kx), (\bigcup \lambda_{A_i})(sy) \} \\ \text{iv)} \quad & (\bigcup \lambda_{A_i})(kx \wedge sy) = \vee \lambda_{A_i}(kx \wedge sy) \leq \vee \max_i \{ \lambda_{A_i}(kx), \lambda_{A_i}(sy) \} \\ & \leq \max_i \{ (\bigcup \lambda_{A_i})(kx), (\bigcup \lambda_{A_i})(sy) \} \end{aligned}$$

Therefore $\bigcup A_i$ is a Contra FL KS operator group of T

Proposition 2.4: P is a Contra FL KS operator group of T. If Q is a fuzzy set in T given by $Q(x) = P(e) - P(x) + 1$ for every x $\in T$. Then Q is a Contra FL KS operator group of T consisting P.

Proof – Consider x, y $\in T$

$$\begin{aligned} \text{i)} \quad & Q(kx sy) = P(e) - P(kx sy) + 1 \leq P(e) - \max_i \{ P(kx), P(sy) \} + 1 \\ & \leq \max_i \{ P(e) - P(kx) + 1, P(e) - P(sy) + 1 \} \leq \max_i \{ Q(kx), Q(sy) \} \\ \text{ii)} \quad & Q((kx)^{-1}) = P(e) - P((kx)^{-1}) + 1 \leq P(e) - P(kx) + 1 \leq Q(kx) \\ & Q((sx)^{-1}) = P(e) - P((sx)^{-1}) + 1 \leq P(e) - P(sx) + 1 \leq Q(sx) \end{aligned}$$

$$\begin{aligned} \text{iii) } Q(kx \vee sy) &= P(e) - P(kx \vee sy) + 1 \leq P(e) - \max\{P(kx), P(sy)\} + 1 \\ &\leq \max\{P(e) - P(kx) + 1, P(e) - P(sy) + 1\} \leq \max\{Q(kx), Q(sy)\} \\ \text{iv) } Q(kx \wedge sy) &= P(e) - P(kx \wedge sy) + 1 \leq P(e) - \max\{P(kx), P(sy)\} + 1 \\ &\leq \max\{P(e) - P(kx) + 1, P(e) - P(sy) + 1\} \leq \max\{Q(kx), Q(sy)\} \end{aligned}$$

Also $P(x) \leq Q(x)$ for all $x \in T$.

Therefore Q is a Contra FL A-B operator group of T consisting P .

REFERENCES:

- [1] Ajmal N and K.V.Thomas, The Lattice of Fuzzy subgroups and fuzzy normal sub groups, Inform. sci. 76 (1994), 1 – 11.
- [2] Birkoff, G : Lattice theory 3rd edn. Amer.Math. Soc. Colloquium pub.25 (1984).
- [3] G.S.V. Satya Saibaba. Fuzzy lattice ordered groups, South east Asian Bulletin of Mathematics 32,749-766 (2008).
- [4] J.A. Goguen : L – Fuzzy Sets, J. Math Anal.Appl. 18, 145-174 (1967).
- [5] L. A.Zadeh : Fuzzy sets, Inform and Control, 8, 338-353 (1965).
- [6] M.Marudai & V. Rajendran: Characterization of Fuzzy Lattices on a Group International Journal of Computer Applications with Respect to T-Norms, 8(8),0975 – 8887 (2010)
- [7] Mordeson and D.S. Malik : Fuzzy Commutative Algebra, World Scientific Publishing Co. Pvt. Ltd.
- [8] Nanda, S : Fuzzy Lattices, Bulletin Calcutta Math. Soc. 81 (1989) 1 – 2.
- [9] Rosenfeld : Fuzzy groups, J. Math. Anal. Appl. 35, 512 – 517 (1971).
- [10] S. Subramanian, R Nagarajan & Chellappa , Structure Properties of M-Fuzzy Groups Applied Mathematical Sciences,6(11),545-552(2012)
- [11] Solairaju and R. Nagarajan : Lattice Valued Q-fuzzy left R – Submodules of Neat Rings with respect to T-Norms, Advances in fuzzy mathematics 4(2), 137 – 145 (2009).
- [12] W.X.Gu. S.Y.Li and D.G.Chen, fuzzy groups with operators, fuzzy sets and system,66 (1994) ,363-3