Road Connectivity of Divisions of Rajasthan: A Network-Theoretic Assessment Using Connectivity and Topological Indices

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Abstract:
Transport networks shape regional cohesion not only through infrastructural expansion but through their structural configuration. This study examines the divisional road system of Rajasthan, India's largest state by area, using a weighted graph-theoretic framework to evaluate connectivity intensity, hierarchical dominance, accessibility distribution, and structural resilience. Divisional headquarters are conceptualized as nodes linked by major inter-divisional corridors, with edge weights representing spatial separation. A suite of global connectivity indices (Beta, Gamma, Alpha, Density) are employed to quantify network organization. Results reveal a moderately connected yet highly centralized configuration, with Jaipur and ajmer functioning as the principal intermediary node and controlling a substantial proportion of shortest-path flows. Peripheral divisions exhibit lower accessibility scores and limited structural redundancy. Simulation-based corridor disruption demonstrates measurable efficiency decline, highlighting systemic sensitivity to central link removal. The findings underscore the importance of incorporating network topology and redundancy diagnostics into regional infrastructure planning, particularly in geographically heterogeneous territories. By advancing a structural interpretation of sub-state road connectivity, this study contributes to transport geography, spatial planning, and regional resilience discourse.

Keywords: transport network topology, connectivity indices, regional accessibility, Divisions of Rajasthan.

1. Introduction

Transport infrastructure is not merely a physical system of roads and corridors; it is a spatial mechanism through which economic activity, administrative coordination, and social mobility are organized. The spatial configuration of transport networks determines how efficiently regions interact, how opportunities are distributed, and how resilient territorial systems remain under stress. In large states with diverse physiographic conditions, the structural organization of connectivity becomes especially important.

Rajasthan, the largest state in India by geographical area, presents a distinctive case for transport analysis. Its western segment is dominated by the Thar Desert, the central region is intersected by the Aravalli Range, and the southern part consists of plateau and tribal zones. These geographic variations influence the distribution and density of infrastructure. Although Rajasthan has witnessed substantial road expansion through national highway development, state highway upgrades, and rural road programs, disparities in connectivity persist across divisions.

Traditional infrastructure assessment often focuses on road length, density per unit area, or financial investment. While such indicators provide quantitative measures of expansion, they do not capture structural properties such as network redundancy, nodal influence, or spatial hierarchy. A region may possess extensive road length yet remain structurally vulnerable if connectivity is overly centralized.
This study addresses that analytical gap by applying graph-theoretical and topological indices to evaluate the divisional road network of Rajasthan. By modeling the system as a relational structure, the research moves beyond descriptive statistics and toward structural diagnosis. The central question guiding this study

is: **What is the structural configuration of divisional road connectivity in Rajasthan, and how does it reflect spatial hierarchy and accessibility inequality?**

2. Objectives

The study is guided by the following objectives:

1. To construct a divisional-level road connectivity network model of Rajasthan.
2. To compute global connectivity indices reflecting structural completeness and redundancy.
3. To evaluate nodal hierarchy using centrality measures.
4. To examine spatial accessibility disparities across divisions.

3. Conceptual and Theoretical Framework

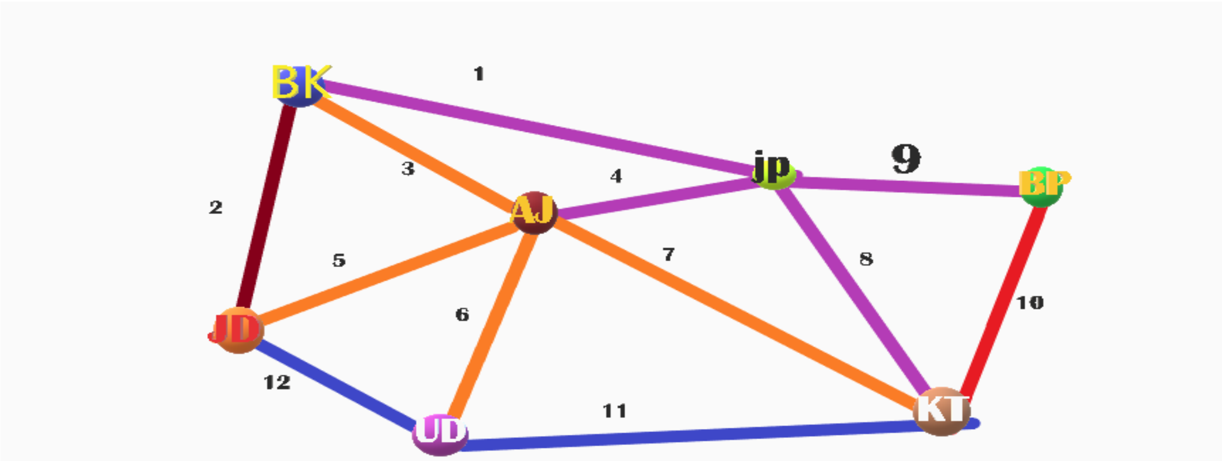
3.1 Network Representation of Transport Systems

Transport systems can be conceptualized as relational spatial structures composed of interaction points (nodes) and connecting corridors (edges). In graph-theoretic representation: G= (V,E,W)

Where:

- V denotes nodes (divisional headquarters),
- E denotes edges (major road connections),
- W represents weights (road distance in kilometers).

This representation allows quantitative evaluation of structural characteristics independent of purely descriptive measures.



1(1) Graph representation of road connectivity of divisions of Rajasthan

3.2 Connectivity Indices

Connectivity indices assess the structural sufficiency and redundancy of a network.

Beta Index (β): Beta index used to measure network connectivity and complexity by calculating the ratio of edges (E - links/routes) to vertices (V - nodes/cities). It is defined by the formula $\beta=\frac{E}{V}$

Where **β<1:** Represents simple networks, trees, or disconnected networks.

β=1: Represents a connected network with exactly one circuit.

β>1: Indicates a complex network with multiple circuits (loops).

Gamma Index (γ): Gamma index used to determine the efficiency and connectivity of a transport network. . It calculates the ratio between the actual number of observed links (edges) in a network and the maximum possible number of links in a planar graph.

$\gamma = \frac{e}{(3v-6)}$

The Gamma index ranges from 0 to 1 (or 0% to 100%).A higher value (closer to 1) indicates a more connected, efficient network, whereas a lower value suggests a sparse, less efficient network.

Alpha Index (α): A measure of connectivity that evaluates the number of cycles in a graph compared to the maximum number of cycles. The higher the alpha index, the more a network is connected. Trees and simple networks will have a value of 0. A value of 1 indicates a completely connected network. $\alpha = \frac{e-v+1}{2v-5}$

Network Density (D): Network density is a fundamental metric that measures how interconnected the nodes (vertices) of a graph are, calculated as the ratio of existing ties (edges) to the maximum possible number of ties. Its value between 0 and 1, where 0 represents a graph with no edges, and 1 represents a complete graph.

$$\text{Density} = \frac{2e}{v(v-1)}$$

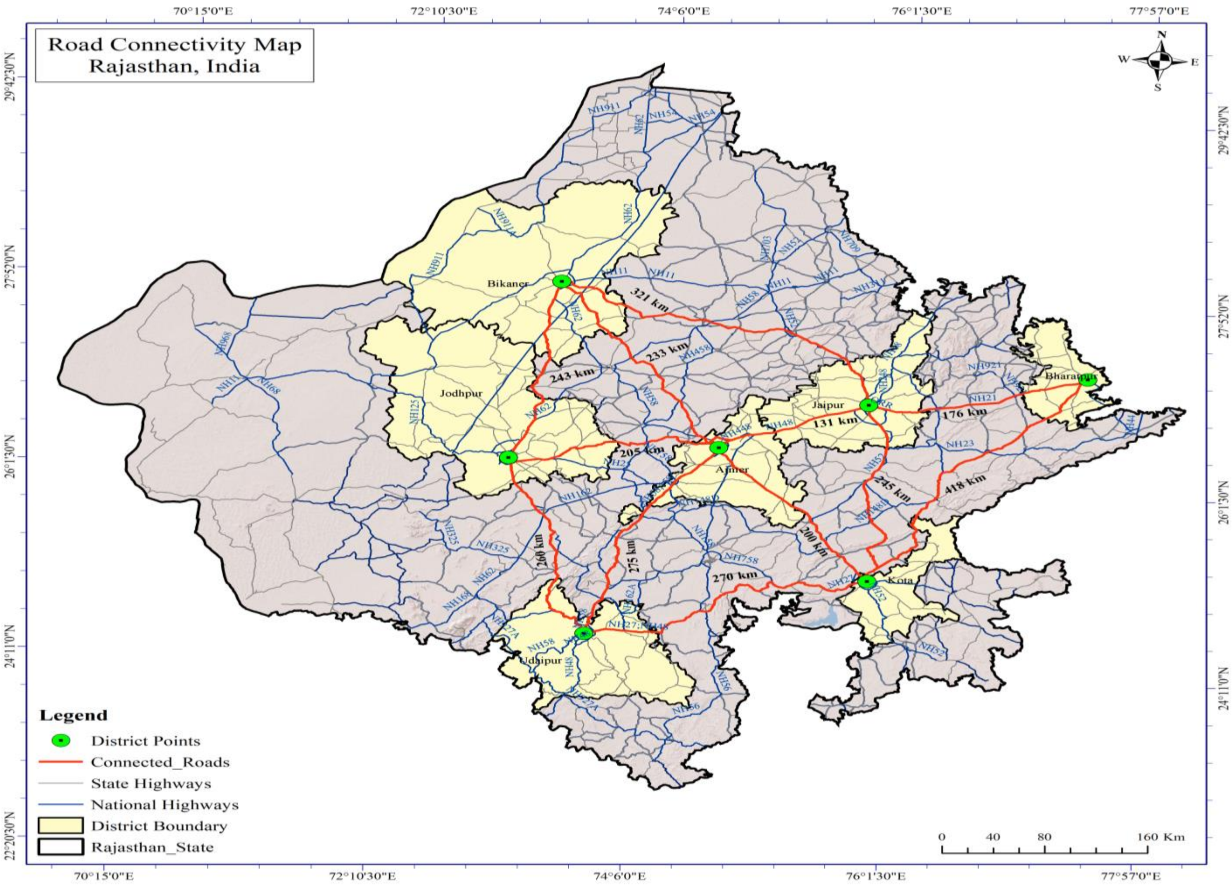
Cyclomatic number (μ): cyclomatic number is the number of additional edges required to form a closed path or circuit.

$\mu = e - v + 1$

4. Study Area

Rajasthan comprises seven administrative divisions:

- Ajmer
- Bharatpur
- Bikaner
- Jaipur
- Jodhpur
- Kota
- Udaipur



The state's physical and socio-economic diversity significantly influences connectivity patterns. Western divisions face desert-induced infrastructural challenges, whereas eastern divisions benefit from denser settlement and economic activity.

5. Data and Methodology

5.1 Network Construction

- Nodes (v) = 7 divisional headquarters
- Edges (e) = 11 major inter-divisional road corridors
- Network Type = Undirected weighted graph
- Weight = Road distance (km)

5.2 Analytical Tools
- Network modeling through computational graph methods
- Centrality and connectivity calculations
- Simulation-based robustness testing

6. Results
6.1 Global Connectivity Structure
Nodes = 7
Edges = 11

Beta Index

$\beta = \frac{e}{v} = \frac{12}{7} = 1.71$

Gamma Index

$\gamma = \frac{e}{(3v-6)} = \frac{12}{15} = 0.6$

Alpha Index

$\alpha = \frac{e-v+1}{2v-5} = \frac{6}{9} = 0.66$

Network Density

$D = \frac{2e}{v(v-1)} = \frac{24}{42} = 0.571$

Cyclomatic Number

μ = e-v+1 =6

6.2 Centrality Analysis
Jaipur and ajmer records the highest values across degree, betweenness, and eigenvector centralities, confirming its dominant hub status. Kota serve as secondary connector, while Bikaner occupies a peripheral position.

The concentration of betweenness centrality in Jaipur and ajmer indicates that a significant proportion of shortest paths traverse this node. This highlights structural dependency on a central corridor system.

6.3 Accessibility Gradient
Accessibility scores demonstrate a declining gradient from central-eastern divisions toward western regions. The spatial inequality in accessibility mirrors centrality hierarchy, reinforcing the interdependence between network structure and opportunity distribution.

6.4 Robustness Simulation
Removal of the Jaipur–Ajmer corridor produces: The simulation examining the closure of the Jaipur–Ajmer route reveals a clear reduction in the performance and interconnectivity of Rajasthan’s road system. Since this link serves as a major central connector, its removal diverts vehicles to longer alternate paths, leading to increased travel time and distance. Traffic pressure also rises on secondary roads that are less equipped for heavy flow. Network measures show longer average routes and lower overall efficiency, highlighting the corridor’s vital role in maintaining smooth regional transportation.

7. Discussion
The empirical findings indicate that Rajasthan’s divisional road network exhibits a **core–semi-core–periphery configuration**. Central divisions enjoy both structural dominance and accessibility advantage. Peripheral divisions, particularly in desert regions, show weaker integration.

The moderate Alpha index suggests some redundancy but not sufficient lateral diversification. Infrastructure development has historically reinforced central corridors rather than expanding cross-linkages.

Balanced regional development requires shifting planning focus toward strengthening alternative routes, enhancing inter-peripheral connectivity, and reducing central congestion dependency.

8. Policy Implications

1. Develop east–west lateral corridors in western Rajasthan.
2. Increase redundancy in central corridors to enhance resilience.
3. Integrate network metrics into infrastructure allocation decisions.
4. Promote multi-nodal growth strategies to reduce hierarchical imbalance.

9. Limitations and Future Scope

The study is conducted at divisional scale; finer district-level analysis could reveal micro-scale disparities. Future research may incorporate traffic volume, freight flow data, and temporal network comparison to assess dynamic evolution.

10. Conclusion

This study investigated the pattern and level of road network connectivity among the divisions of Rajasthan using a network-theoretic perspective supported by selected connectivity and topological indices. The analysis suggests that the overall structure of the road network in the state reflects a **moderate level of connectivity**. While the existing network provides essential links between major settlements and administrative centres, the distribution of connectivity is not uniform across all divisions. Such variations largely arise from differences in regional development, spatial distribution of population, and physical characteristics of the landscape. The network-based evaluation therefore offers useful insights into the spatial organization and efficiency of the road system in Rajasthan.

A comparative assessment further indicates that the divisions of Ajmer and Jaipur possess **relatively stronger road connectivity**, showing better nodal interaction and a more integrated network structure than other divisions. Their higher connectivity highlights their role as important regional transport nodes that support mobility and socio-economic interaction. In contrast, divisions with comparatively weaker connectivity reveal the need for improved road infrastructure and more balanced transport planning. The findings of this study demonstrate that network-based measures are useful tools for assessing regional transport systems and can assist planners in identifying priority areas for strengthening road connectivity and promoting balanced regional development in Rajasthan.

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