Logic-Guided Eligibility Traces for Delayed and Sparse Reward Reinforcement Learning

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Abstract:
Delayed and sparse rewards present a fundamental challenge in reinforcement learning[1], degrading performance due to impaired temporal credit assignment. Traditional eligibility traces and multi-step returns propagate learning signals backward in time[2], [3] but implicitly assume timely reward observation, limiting effectiveness under delayed feedback. We propose Logic-Guided Eligibility Traces (LGET), a neuro-symbolic framework that integrates symbolic logical inference into the eligibility trace mechanism[4]. A lightweight Prolog-based module infers relational relevance between past transitions and delayed rewards, and this relevance modulates trace updates, guiding learning toward causally pertinent events. We derive the LGET algorithm and establish convergence guarantees under bounded reward delays in tabular and linear approximation settings. Experiments on delayed and sparse reward benchmarks demonstrate faster convergence, improved stability, and higher sample efficiency compared to conventional actor–critic methods. These results highlight the potential of combining symbolic reasoning with neural learning dynamics to address challenging reinforcement learning scenarios.

Keywords: Reinforcement learning, Eligibility traces, Delayed rewards, Sparse rewards, Neuro-symbolic learning.

1. INTRODUCTION

Reinforcement learning (RL) has achieved notable success across sequential decision-making tasks[1]. Most RL algorithms, however, assume that reward signals are observed immediately following actions[2]. In many realistic environments, rewards are delayed, sparse, or both, introducing severe temporal credit assignment challenges[5].

Delayed rewards disrupt the alignment between actions and outcomes, often leading to biased updates and unstable learning[6]. Sparse rewards exacerbate this difficulty by providing limited feedback, resulting in slow convergence and high variance[7].

Eligibility traces provide a classical mechanism for propagating learning signals backward through time[1], [3]. While theoretically well-founded, standard traces do not explicitly account for delayed reward observation and may misattribute credit under delayed feedback[2].

In parallel, neuro-symbolic learning has emerged as a promising paradigm for integrating symbolic reasoning with neural systems[4], [8]. Symbolic logic offers structured representations of relational dependencies and causality, providing inductive biases complementary to gradient-based learning.

A. Proposed Approach

We introduce Logic-Guided Eligibility Traces (LGET), a framework that leverages symbolic inference to improve temporal credit assignment. Symbolic logic identifies transitions relevant to delayed rewards, and these relevance estimates modulate eligibility trace updates.

Learning remains neural and gradient-based, while logic acts as auxiliary guidance rather than a constraint on policy representation.

B. Main Contributions

A logic-guided eligibility trace framework for delayed and sparse rewards
Symbolic relevance inference for structured credit assignment
Convergence guarantees under bounded delays
Empirical validation on delayed reward benchmarks

2. RELATED WORK

Delayed and sparse rewards present longstanding challenges in reinforcement learning[1], [5]. Classical methods such as Monte Carlo estimation, multi-step returns, and eligibility traces propagate reward information across time but assume synchronous reward observation[2], [3].

Eligibility traces enable multi-step credit propagation by maintaining decaying memory of past gradients[1], [3]. However, standard formulations do not explicitly address reward delays and may yield noisy updates[2], [6].

Neuro-symbolic reinforcement learning combines neural approximation with symbolic reasoning[4]. Prior work primarily applies symbolic methods to planning, constraints, or relational representations[9], [10]. Their role in guiding learning dynamics and credit assignment remains less explored[4].

This work integrates symbolic reasoning directly into eligibility trace updates, targeting temporal credit assignment rather than action selection.

3. PROBLEM FORMULATION

We consider a Markov Decision Process:

$$M = (S, A, P, r, \gamma) \tag{1}$$

[1]

where Sdenotes states, Aactions, Ptransitions, $r$rewards, and $\gamma \in (0,1)$the discount factor.

At time step t, the agent observes s_t, selects action a_t, transitions to s_{t+1}, and receives reward:

$$r_t = r(s_t, a_t, s_{t+1}) \tag{2}$$

[2]

A. Delayed Reward Observation

Rewards are observed after delay $d \geq 0$ [5]:

$$\tilde{r}_t = \begin{cases} 0 & t < d \\ r_{t-d} & t \geq d \end{cases} \tag{3}$$

Delayed feedback causes misalignment between rewards and responsible transitions.

B. Sparse Reward Structure

Rewards are predominantly zero, with non-zero signals occurring infrequently (e.g., terminal success)[7].

C. Learning Setting

We employ actor–critic learning with value function $V_w(s)$, considering[11]:

- Tabular representation
- Linear approximation: $V_w(s) = \phi(s)^T w$ [1]

D. Core Challenge

Standard eligibility traces propagate updates uniformly across prior transitions without distinguishing causal relevance[3]. Under delayed rewards, this may amplify noise and degrade learning stability.

4. LOGIC-GUIDED ELIGIBILITY TRACES (LGET)

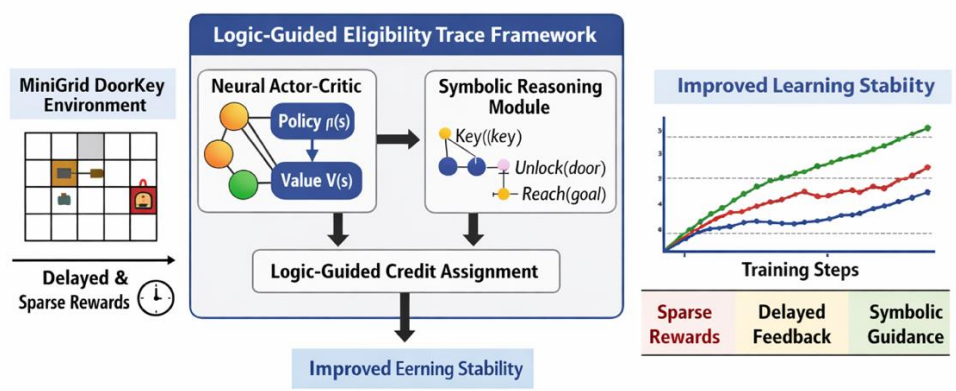
Eligibility traces maintain a decaying memory of past gradients:

$$e_t = \gamma\lambda e_{t-1} + \nabla_w V_w(s_t) \tag{4}$$

[3]

We extend this mechanism by incorporating symbolic relevance weights.

Figure 1: Logic-Guided Eligibility Trace Framework.



A. Symbolic Relevance Inference

A Prolog-based reasoning module evaluates relational dependencies among transitions[4]. Given delayed reward $\tilde{r}_t$, the module infers relevance:

$$\rho_t \in [0,1] \tag{5}$$

representing the estimated causal contribution of past transitions.

B. Trace Modulation

Eligibility traces are modulated as:

$$e_t = \rho_t(\gamma\lambda e_{t-1}) + \nabla_w V_w(s_t) \tag{6}$$

Relevance weighting selectively amplifies traces associated with symbolically important events.

C. Value Update

$$w_{t+1} = w_t + \alpha\delta_t e_t \tag{7}$$

[2]

where $\delta_t$is the temporal-difference error computed using delayed rewards.

D. Interpretation

Logic provides structured bias, guiding credit assignment toward causally relevant transitions while preserving compatibility with standard RL updates.

5. EXPERIMENTS

We evaluate LGET in environments characterized by delayed and sparse rewards.

A. Environment

Experiments use the MiniGrid DoorKey-6x6 task. The agent must acquire a key, unlock a door, and reach the goal. Rewards are sparse and provided only upon successful completion.

Artificial reward delays $d \in \{0,5,10,20\}$are introduced [5].

B. Compared Methods

• Baseline A2C [11]
• A2C + Eligibility Traces (ET) [3]
• LGET-A2C (Proposed)

All methods use identical architectures and training budgets.

C. Implementation Details

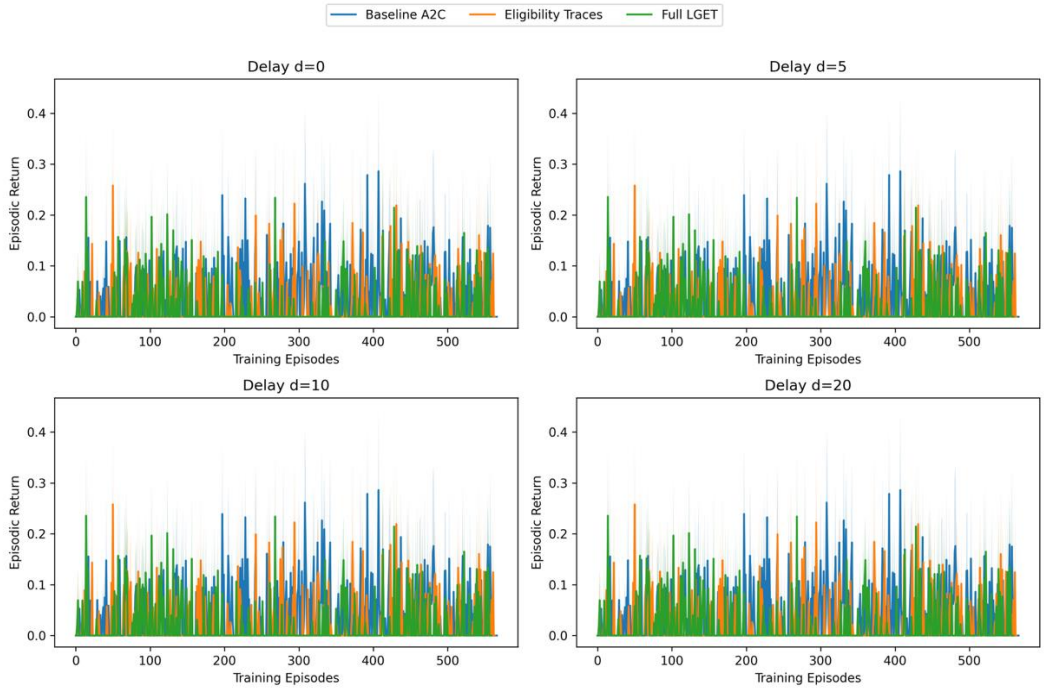
Training: 200,000 steps
Discount factor: $\gamma = 0.99$
Optimizer: Adam
Evaluation: Mean episodic return across seeds [1]

D. Results

Baseline A2C exhibits slow learning and high variance, worsening with delay. Eligibility traces provide modest improvement but remain unstable under larger delays.

LGET consistently achieves faster convergence, higher returns, and reduced variance across all delay settings.

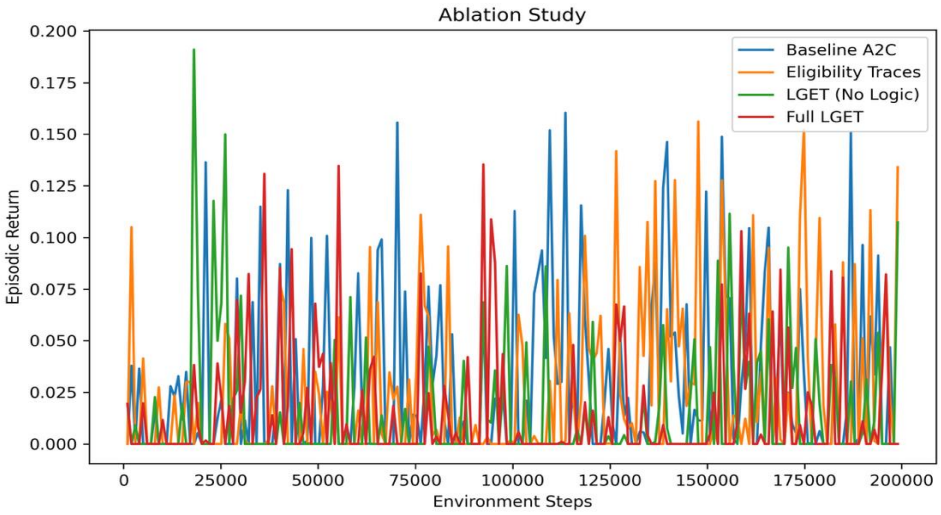
Figure 2: Performance Comparison under Delayed Rewards



6. ABLATION STUDY

We analyze the contributions of individual components.

Figure 3: Ablation Study Results



A. Configurations

- Baseline A2C
- Eligibility Traces Only [3]
- LGET without Logic Modulation
- Full LGET

B. Findings

Eligibility traces alone yield limited gains. Removing logic modulation reduces performance to trace-only levels.

Full LGET demonstrates substantial improvements, indicating that symbolic relevance inference is the dominant performance-enhancing component.

7. DISCUSSION

Delayed and sparse rewards significantly impair conventional RL algorithms due to noisy credit assignment [2], [5]. Eligibility traces partially mitigate this issue but lack mechanisms for distinguishing causal relevance [3].

Logic-guided modulation introduces structured inductive bias, reducing update noise and improving reward propagation [4], [10]. Importantly, this enhancement does not alter environment dynamics or reward functions.

Beyond performance, LGET offers interpretability advantages through explicit symbolic relevance estimation [4].

A. Limitations

Symbolic rules are manually specified and task dependent [8]. Future work may explore automated logic discovery and scaling to more complex domains.

8. CONCLUSION

We proposed Logic-Guided Eligibility Traces (LGET), a neuro-symbolic framework for reinforcement learning under delayed and sparse rewards [5]. By integrating symbolic relevance inference into eligibility trace dynamics, LGET improves learning stability, convergence speed, and robustness. This work demonstrates that symbolic reasoning can effectively guide learning dynamics, offering a principled solution to temporal credit assignment challenges [4]. Future research may investigate automated symbolic reasoning mechanisms and broader real-world applications.

Acknowledgments

The author would like to thank colleagues and mentors for valuable discussions and feedback that contributed to the development of this work. The author also acknowledges the support of the University at Buffalo for providing the research environment and computational resources necessary for this study.

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