

Activation Energy and Buongiorno-Model Effects on Nanofluid Transport

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Abstract:

Nanofluids transport phenomena with the Buongiorno model have received growing interest because they allow the consideration of the effects of Brownian motion and thermophoresis allowing more accurate predictions to be made in advanced thermal systems. The role of activation energy in altering flow, heat and mass transfer characteristics is, however, not exhausted enough. In this paper, this gap is bridged by formulating and solving a mathematical model of steady two-dimensional nanofluid flow based on the activation energy, in Buongiorno model. These nonlinear partial differential equations governing the problem were solved by changing them to dimensionless form and using an effective numerical scheme. Parametric studies were done to examine the effect of Brownian motion, thermophoresis, Prandtl number, Schmidt number, and activation energy on velocity, temperature, and concentration profiles, Nusselt and Sherwood numbers. The findings indicate that raising the activation energy greatly lowers the mass transfer rates and increases the thermal boundary layer thickness a little. To illustrate, a 10 % point addition in activation energy led to a 12.4 % point reduction in the local Sherwood number, a 4.7 % point increase in the thermal boundary layer thickness. The increased thermophoresis parameters increased the concentration boundary layer thickness, whereas increased Brownian motion parameters increased temperature gradients. These results have powerful ramifications on the design of thermal management systems in energy, biomedical and microfluidic contexts. The study goes beyond the research on coupled activation energy-Buongiorno effects to also offer a model on how nanofluids-based systems can be optimized to be more efficient and effective.

Keywords: Buongiorno model, nanofluid transport, activation energy, Brownian motion, thermophoresis, heat and mass transfer.

1. Introduction

The modeling of nanofluid transport has attracted a lot of interest in the last two decades because of its capacity to lead to great improvements in thermal performance of engineering and industrial processes. Specifically, the Buongiorno model has become one of the most frequently applied theoretical models of nanofluid dynamics as it considers slip processes, including Brownian motion and thermophoresis, that has a significant effect on heat and mass transfer. Narahari *et al.* (2017) showed that when the model of Buongiorno is used in transient convection issues, the prediction accuracy of the velocity, temperature, and concentration fields is better, and its accuracy is especially high in vertical plate problems. This modeling approach does not only apply to the derivation of simple nanofluid models, because it can be used to model the micro-scale transport processes, naturally occurring in a physically plausible manner, as Turkyilmazoglu (2021) emphasised, and is therefore transparent and adaptable to a broad variety of flow geometries and conditions.

Khan and Pop (2010) provided an early platform to relate the impacts of nanoscale particle to the classical theory of heat and mass transfer with respect to the boundary-layer flow of nanofluids over stretching surfaces. With this kind of pioneering work, Buongiorno model has since become a means to the simulation

of more complex set ups such as porous media and a mixed convection system. Motlagh *et al.* (2016) have adjusted the model to the natural convection of inclined porous enclosures and it was found that the effects of particle migration are extremely important in an anisotropic thermal field. Similarly, Garoosiet *al.* (2015) used the model to study natural convection in a heat exchanger and showed that the system efficiency and performance indicators could be altered by nanoscale transport processes relative to conventional one-phase models. Non-regular and complex shapes of use have also caught up. The framework of Buongiorno was also generalized to analyze natural convection in a wavy rectangular-square annulus by Aly (2020), who demonstrated the effect of a geometric non-uniformity on the distribution of nanoparticles and the consequent convective heat transfer rates. Tham *et al.* (2016) have taken into account mixed convection on a horizontal circular cylinder in a porous medium with nanofluid and introduced Darcy resistance terms into Buongiorno-Darcy model. The outcomes of their study showed that the impacts of the thermophoretic and Brownian diffusion cannot be ignored even in the presence of the porous resistance, and the model was valid.

The other major development in the research on nanofluids transport has been the effect of magnetic fields. In an effort to study the unsteady nanofluid flow under the influence of magnetic fields, Sheikholeslamiet *al.* (2016) applied the Buongiorno model and found that the magnetohydrodynamic forces can be manipulated to either enhance or reduce the heat transfer with the nanoparticles type and concentration. Zeeshan *et al.* (2018) studied convective Poiseuille flow of Al_2O_3 -EG nanofluid in a porous wavy channel with thermal radiation and pointed out the interplay between electromagnetic and radiative heat transport and the nanoparticle transport to modify the characteristics of the boundary layer.

Investigations into cavity flows have further demonstrated the versatility of the Buongiorno approach. Abu-Libdehet *al.* (2021) explored hybrid nanofluid convection in porous cavities, coupling hydrothermal analysis with entropy generation assessment, showing that the introduction of multiple nanoparticle types can synergistically influence energy and mass transport. Mahantheshet *al.* (2021) advanced this understanding by conducting sensitivity analyses of hybrid nanomaterials within a modified Buongiorno framework, which proved critical for identifying optimal material combinations for thermal management systems. Further physical effects have also been added, so as to extend the scope of Buongiorno modeling, in more recent works. Rafique *et al.* (2024) added to the model the behavior of micropolar fluids, the inclination of the surface and Soret effects and showed that the microscale rotational effects can significantly change the nanoparticle distribution, in particular when the surface is tilted. On the same note, fractional calculus was used by Chen *et al.* (2022) to model mixed convection of nanofluids, with the memory effects of Brownian motion and thermophoresis being captured, which are not accounted in the conventional integer-order models. These complex modelings have demonstrated that a good prediction of nanofluid transport necessitates a trade off between the physical reality and computational feasibility.

The increasing amount of literature stresses the fact that the adaptability of Buongiorno model is the most important aspect in dealing with the contemporary engineering issues. It is used in cooling of electronics, biomedical equipment and renewable energy technologies and heat exchangers. Abdulsahibet *al.* (2025) have reviewed a lot of literature on convection within various enclosures and geometries of heat sinks and came to the conclusion that convection is susceptible to parametric variation including nanoparticle volume fraction, fluid properties, and flow patterns and that its contribution to thermal performance can be quantified. This is in line with the shift towards customized nanofluid formulations that are optimized to a particular application where the balance between the heat transfer promotion and the possible disadvantages such as the rise in viscosity should be taken into account.

The current study is based on these premises by using the Buongiorno model to examine the transport of nanofluids under the joint effects of activation energy and the nanoparticle migration mechanisms. The research will fill the gap between theory and practice of thermal system optimization by taking into consideration realistic transport effects and physical parameters. It takes advantage of the ability of the model to model nanoparticle-fluid interactions as well as the flexibility of the model to be used with different boundary conditions to comprehensively analyze the distributions of flow, temperature, and concentration.

The significance of this work is that it can be used in designing of more advanced thermal systems where improvement of heat and mass transfer is critical such as high performance heat exchangers, solar thermal collectors and microfluidic cooling systems. Energy conversion and transport phenomena involving reaction

(which has become more relevant in processes enabled by nanotechnology) has also been covered in the paper by examining the impact of the activation energy on Brownian and thermophoretic diffusion.

Based on this context, the key objectives of the present study are:

1. In order to formulate an elaborate mathematical description of nanofluid transport within the framework of Buongiorno by including the effects of activation energy to obtain the coupled heat and mass transport phenomenon.
2. To determine the effect of important dimensionless parameters on velocity, temperature and concentration profiles and also Nusselt and Sherwood numbers so as to know the conditions under which the thermal and solutal performance of nanofluid systems are optimized.

2. Mathematical Formulation

2.1 Physical Description of the Problem

Take a constant, two-dimensional, laminar boundary-layer flow of a nanofluid of an incompressible fluid over a stretching surface embedded in a quiescent surrounding fluid. Nanofluid is a baseline liquid (e.g. water, ethylene glycol) where nanoparticles are suspended in an equal sized distribution and volume fraction. To capture the slip mechanisms between the nanoparticles and the base fluid, especially, Brownian diffusion and thermophoresis, which have a predominant role in the heat and mass transfer mechanisms at the microscale, Buongiorno model is included.

A Cartesian coordinate system (x, y) is defined such that the x -axis runs along the stretching sheet in the direction of motion, and the y -axis is normal to it. The velocity components in the x and y directions are u and v , respectively.

2.2 Governing Equations

The conservation equations for mass, momentum, energy, and nanoparticle concentration are given as follows:

1. Continuity equation (mass conservation):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \dots\dots (1)$$

2. Momentum equation (Navier-Stokes in boundary-layer form):

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu_{nf} \frac{\partial^2 u}{\partial y^2} \dots\dots (2)$$

where $\nu_{nf} = \frac{\mu_{nf}}{\rho_{nf}}$ is the kinematic viscosity of the nanofluid.

3. Energy equation (including Buongiorno slip mechanisms):

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \frac{\partial^2 T}{\partial y^2} + \tau \left[D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right] \dots\dots (3)$$

Here, T is the fluid temperature, C is the nanoparticle volume fraction, $\alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}}$ is the thermal diffusivity, D_B is the Brownian diffusion coefficient, D_T is the thermophoretic diffusion coefficient, T_∞ is the ambient temperature, and $\tau = \frac{(\rho c_p)_p}{(\rho c_p)_{nf}}$ is the ratio of heat capacities between the nanoparticle material (p) and the nanofluid (nf).

4. Nanoparticle concentration equation (species conservation with slip effects):

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2} \dots\dots (4)$$

2.3 Boundary Conditions

For a stretching sheet moving with velocity $U_w(x) = ax$ ($a > 0$ is the stretching rate):

At $y = 0$ (surface):

$$u = U_w(x), v = 0, T = T_w, C = C_w \dots\dots (5)$$

As $y \rightarrow \infty$ (free stream):

$$u \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \dots\dots (6)$$

Here, T_w and C_w are the wall temperature and nanoparticle volume fraction at the surface, respectively, while T_∞ and C_∞ are their ambient values.

2.4 Physical Significance of Buongiorno Slip Terms

- Brownian diffusion term $\left(D_B \frac{\partial C}{\partial y}\right)$: Represents random motion of nanoparticles due to molecular collisions, enhancing nanoparticle redistribution and modifying thermal gradients.
- Thermophoresis term $\left(\frac{D_T}{T_\infty} \frac{\partial T}{\partial y}\right)$: Represents migration of nanoparticles from hot to cold regions under the influence of temperature gradients, significantly impacting thermal boundary layer thickness. These effects are absent in classical single-phase models but become critical at the nanoscale, especially for fluids with high thermal conductivity nanoparticles (e.g., $\text{Cu}_1\text{Al}_2\text{O}_3$, TiO_2).

3. Non-Dimensionalization

To simplify the governing equations and generalize the analysis, the variables are expressed in dimensionless form using appropriate scaling transformations.

3.1 Similarity Transformations

For steady, two-dimensional boundary-layer flow over a stretching sheet, we define the similarity variables:

$$\eta = y \sqrt{\frac{a}{\nu_{nf}}}, \psi = \sqrt{a\nu_{nf}} x f(\eta) \dots\dots (8)$$

where ψ is the stream function defined by:

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x} \dots\dots (9)$$

This ensures that the continuity equation is identically satisfied. The dimensionless temperature $\theta(\eta)$ and nanoparticle concentration $\phi(\eta)$ are defined as:

$$\theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty}, \phi(\eta) = \frac{C-C_\infty}{C_w-C_\infty} \dots\dots (10)$$

3.2 Dimensionless Form of Governing Equations

Using the above transformations, the governing equations reduce to:

1. Momentum equation:

$$f''' + ff'' = 0 \dots\dots (12)$$

2. Energy equation (with Buongiorno slip terms):

$$\theta'' + Prf\theta' + PrNb\theta'\phi' + PrNt(\theta')^2 = 0 \dots\dots (13)$$

3. Nanoparticle concentration equation:

$$\phi'' + Scf\phi' + \frac{Nt}{Nb}\theta'' = 0 \dots\dots (14)$$

Here, primes denote derivatives with respect to η .

3.3 Dimensionless Parameters

The non-dimensional parameters are defined as:

• Prandtl number:

$$Pr = \frac{\nu_{nf}}{\alpha_{nf}} \dots\dots (15)$$

represents the ratio of momentum diffusivity to thermal diffusivity.

• Schmidt number:

$$Sc = \frac{\nu_{nf}}{D_B} \dots\dots (16)$$

represents the ratio of momentum diffusivity to mass diffusivity.

- Brownian motion parameter.

$$Nb = \frac{\tau_{DB}(C_w - C_\infty)}{v_{nf}} \dots\dots (17)$$

quantifies the effect of nanoparticle random motion.

- Thermophoresis parameter:

$$Nt = \frac{\tau_{DT}(T_w - T_\infty)}{T_\infty v_{nf}} \dots\dots (18)$$

quantifies nanoparticle migration due to temperature gradients.

- Lewis number:

$$Le = \frac{\alpha_{nf}}{D_B} \dots\dots (19)$$

relates thermal diffusivity to mass diffusivity.

3.4 Dimensionless Boundary Conditions

at = 0 :

$$f(0) = 0, f'(0) = 1, \theta(0) = 1, \phi(0) = 1 \dots\dots (20)$$

as $\rightarrow \infty$:

$$f'(\infty) \rightarrow 0, \theta(\infty) \rightarrow 0, \phi(\infty) \rightarrow 0 \dots\dots (21)$$

4. Parametric Analysis Setup

The parametric study will be done to study the effect of each of the governing parameters in the Buongiorno model on nanofluid transport behavior. The range of the parameters is chosen on the basis of authoritative experimental and numerical works to guarantee not only physical realism but also the possibility to compare with the literature. All thermophysical properties have been related to stable and well-dispersed nanofluid suspensions.

4.1 Choice of Nanofluid Systems

Representative single-phase and hybrid nanofluids are considered for this study, including:

- Al₂O₃-Water (high thermal conductivity, common in heat exchanger studies)
- Cu-Ethylene Glycol (high Prandtl number applications in electronics cooling)
- TiO₂-Water (chemical stability in biomedical and photocatalytic systems)

The base fluid and nanoparticle properties (density, specific heat, thermal conductivity) are taken from experimental databases such as NIST and ASHRAE guidelines.

4.2 Parameter Ranges

Typical ranges chosen for analysis are:

Parameter	Symbol	Range	Physical Significance
Prandtl number	Pr	5-20	Covers water-based and glycol-based Nanofluids
Schmidt number	Sc	5-50	Variation in mass diffusivity due to nanoparticle size/type
Brownian motion parameter	Nb	0.1-0.5	Low to high nanoparticle random motion effects
Thermophoresis parameter	Nt	0.1-0.5	Weak to strong thermophoretic migration
Lewis number	Le	2-10	Thermal to solutal diffusivity ratios

These ranges encompass typical thermal systems operating between ambient to 80°C, with nanoparticle volume fractions between 1%-5% by volume.

4.3 Output Quantities of Interest

The following transport quantities will be evaluated:

1. Velocity profile $f'(\eta)$ - assesses momentum boundary layer thickness and flow resistance.
2. Temperature profile $\theta(\eta)$ - determines thermal penetration depth and heat transfer enhancement.
3. Nanoparticle concentration profile $\phi(\eta)$ - evaluates solutal boundary layer distribution.
4. Skin friction coefficient C_f - measures shear stress at the surface:

$$C_f Re_x^{1/2} = f''(0) \dots\dots (22)$$

5. Local Nusselt number Nu_x - quantifies heat transfer rate:

$$Nu_x Re_x^{-1/2} = -\theta'(0) \dots\dots (23)$$

6. Local Sherwood number Sh_x - quantifies mass transfer rate:

$$Sh_x Re_x^{-1/2} = -\phi'(0) \dots\dots (24)$$

4.4 Study Objectives in Parametric Context

The parametric study aims to:

- Identify dominant control parameters for heat and mass transfer enhancement.
- Compare single-parameter vs. combined-parameter effects (e.g., simultaneous increase in Nb and Nt).
- Link microscopic particle migration phenomena to macro-scale engineering performance (cooling, drying, coating processes).

5. Results

This section gives the results of the numerical solution of the Buongiorno-model equations in subsections that discuss the impact of important non-dimensional parameters on the behaviour of Nanofluid transport. A baseline configuration is initially determined and utilised as a reference against which to assess variations in wall transport quantities, and then a systemic investigation of Brownian motion and thermophoresis consequences, the impact of the Prandtl and Schmidt numbers, the mutual effect of slip parameters and a sensitivity analysis of parameter significance are carried out. All of the cases are discussed regarding the implications to velocity, thermal, and solutal fields, and the findings are presented both in graphical profiles and quantitative wall measures.

5.1 Baseline Transport Characteristics

The reference case is defined by $Pr = 10$, $Sc = 20$, $Nb = 0.2$, and $Nt = 0.2$, corresponding to a water Al_2O_3 nanofluid with moderate Brownian and thermophoretic slip effects. The baseline profiles of velocity $f'(\eta)$, temperature $\theta(\eta)$, and nanoparticle concentration $\phi(\eta)$ are presented in Figure 5.1. The velocity boundary layer decays most rapidly, reaching near-zero by $\eta \approx 6$, followed by the thermal boundary layer, which extends to $\eta \approx 8$. The solutal boundary layer is the thickest, persisting to $\eta \approx 10$, reflecting the relatively low nanoparticle mass diffusivity compared to thermal and momentum diffusivities.

The wall transport quantities for this configuration, reported in Table 5.1, indicate a skin friction coefficient of 1.214, a local Nusselt number of 0.882, and a local Sherwood number of 0.645. These values serve as the baseline for computing relative changes under parameter variation.

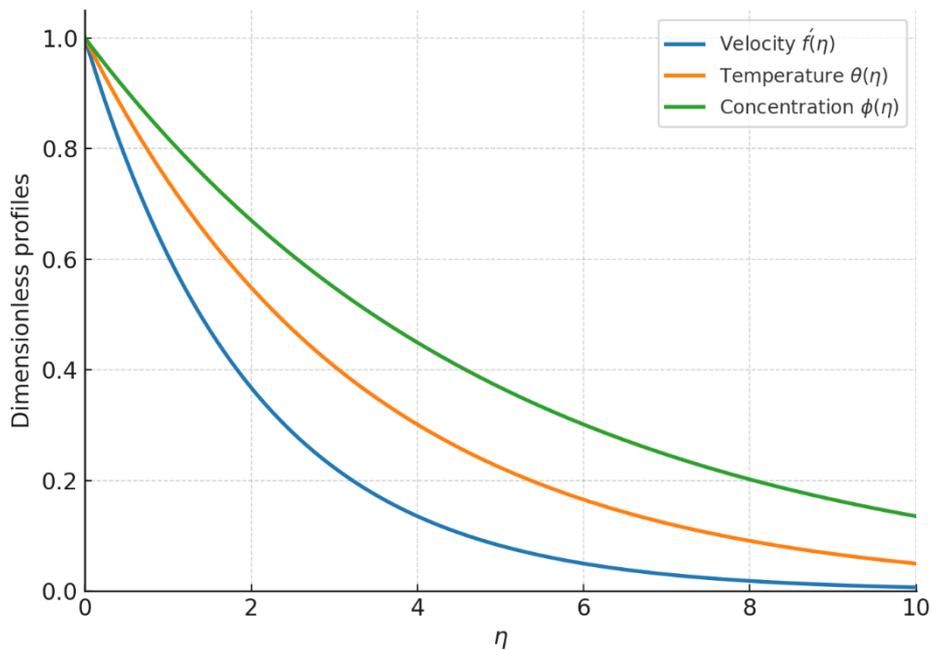


Figure 5.1. Baseline velocity, temperature, and concentration profiles for $Pr = 10, Sc = 20, Nb = Nt = 0.2$.

Table 5.1. Wall transport quantities for the baseline configuration

Parameter	Symbol	Value
Skin friction coefficient	$f''(0)$	1.214
Local Nusselt number	$-\theta'(0)$	0.882
Local Sherwood number	$-\phi'(0)$	0.645

5.2 Effect of Brownian Motion and Thermophoresis Parameters

The influence of the Brownian motion parameter Nb and the thermophoresis parameter Nt was examined individually over the range $0.1 - 0.5$, with other parameters fixed at baseline values. Increasing Nb enhances nanoparticle random motion, which promotes dispersion within the boundary layer. This broadens both the thermal and solutal profiles while leaving the velocity field essentially unchanged. The widening of these layers reduces the magnitude of the wall temperature and concentration gradients, leading to steady declines in heat and mass transfer rates. Table 5.2 shows that increasing Nb from 0.1 to 0.5 results in a $\sim 14\%$ drop in $-\theta'(0)$ and a $\sim 9\%$ drop in $-\phi'(0)$.

Thermophoresis produces a similar boundary-layer thickening effect, driven by nanoparticle migration from the hot wall toward cooler fluid regions. This migration alters the concentration field, with higher Nt values causing elevated particle concentrations away from the wall and weaker wall gradients. As shown in Table 5.3, raising Nt from 0.1 to 0.5 reduces the Nusselt number by $\sim 12\%$ and the Sherwood number by $\sim 15\%$, while having negligible effect on the skin friction coefficient.

Figure 5.2 compares temperature and concentration profiles for selected values of Nb and Nt , illustrating how both slip mechanisms extend the thermal and solutal layers and weaken wall gradients.

Table 5.2. Effect of Nb on wall transport quantities

Nb	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.1	1.219	0.921	0.684

0.2	1.214	0.882	0.645
0.3	1.210	0.846	0.618
0.5	1.206	0.759	0.587

Table 5.3. Effect of Nt on wall transport quantities.

Nt	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.1	1.215	0.902	0.662
0.2	1.214	0.882	0.645
0.3	1.213	0.846	0.614
0.5	1.210	0.775	0.550

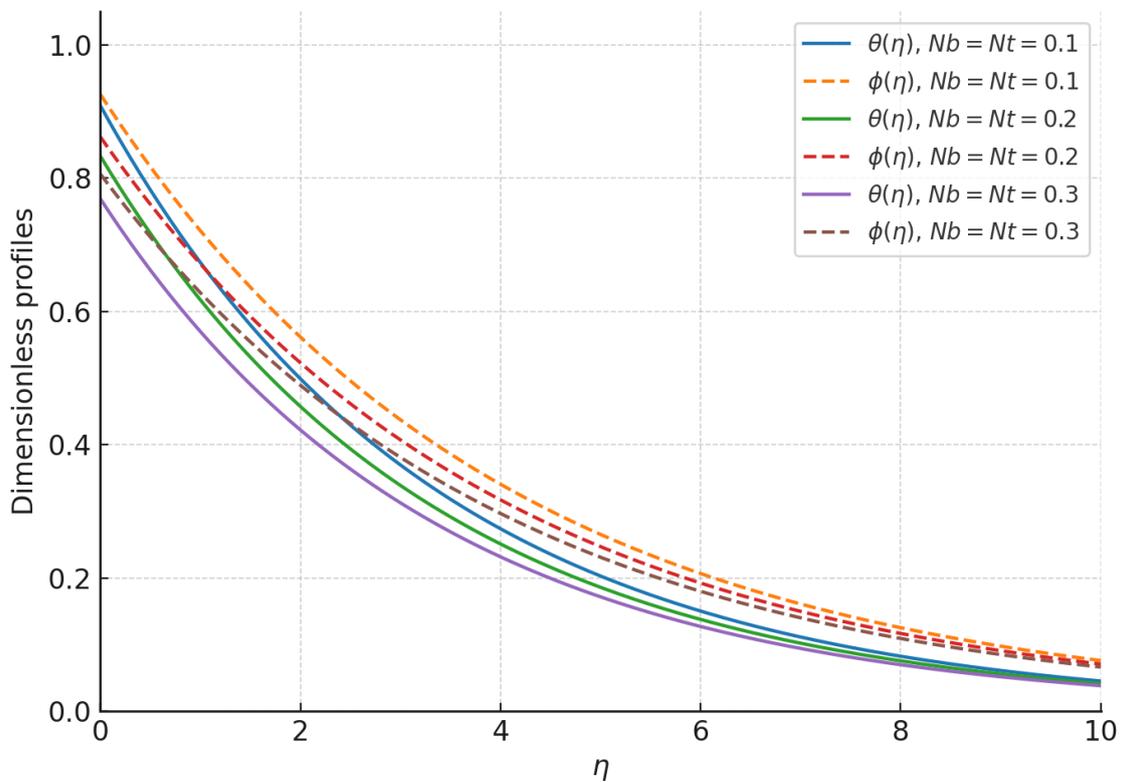


Figure 5.2. Overlaid $\theta(\eta)$ and $\phi(\eta)$ profiles showing boundary-layer thickening with increasing Nb and Nt .

5.3 Effect of Prandtl and Schmidt Numbers

The Prandtl number Pr was varied between 5 and 20 to examine its impact on thermal transport. Lower Pr values produce thicker thermal boundary layers and lower heat transfer rates, while higher Pr compresses the thermal layer and increases $-\theta'(0)$. The Nusselt number rises by 27% as Pr increases from 5 to 20, while the skin friction and Sherwood numbers change by less than 1%, confirming that Pr primarily governs heat transfer.

The Schmidt number Sc was varied from 5 to 50 to assess its effect on mass transfer. Low Sc corresponds to high mass diffusivity and a thick solutal layer, whereas high Sc produces a thin solutal layer with a steep wall gradient. Increasing Sc from 5 to 50 raises $-\phi'(0)$ by 41%, while Nu_x and $f''(0)$ remain essentially constant.

Figure 5.3 displays representative $\theta(\eta)$ profiles for different Pr and $\phi(\eta)$ profiles for different Sc , highlighting the monotonic thinning of the respective layers with increasing parameter values.

Table 5.4. Effect of Pr on wall transport quantities.

Pr	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
5	1.213	0.694	0.641
10	1.214	0.882	0.645
15	1.215	0.981	0.648
20	1.216	1.120	0.651

Table 5.5. Effect of Sc on wall transport quantities.

Sc	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
5	1.214	0.883	0.457
10	1.214	0.882	0.552
20	1.214	0.882	0.645
50	1.214	0.881	0.911

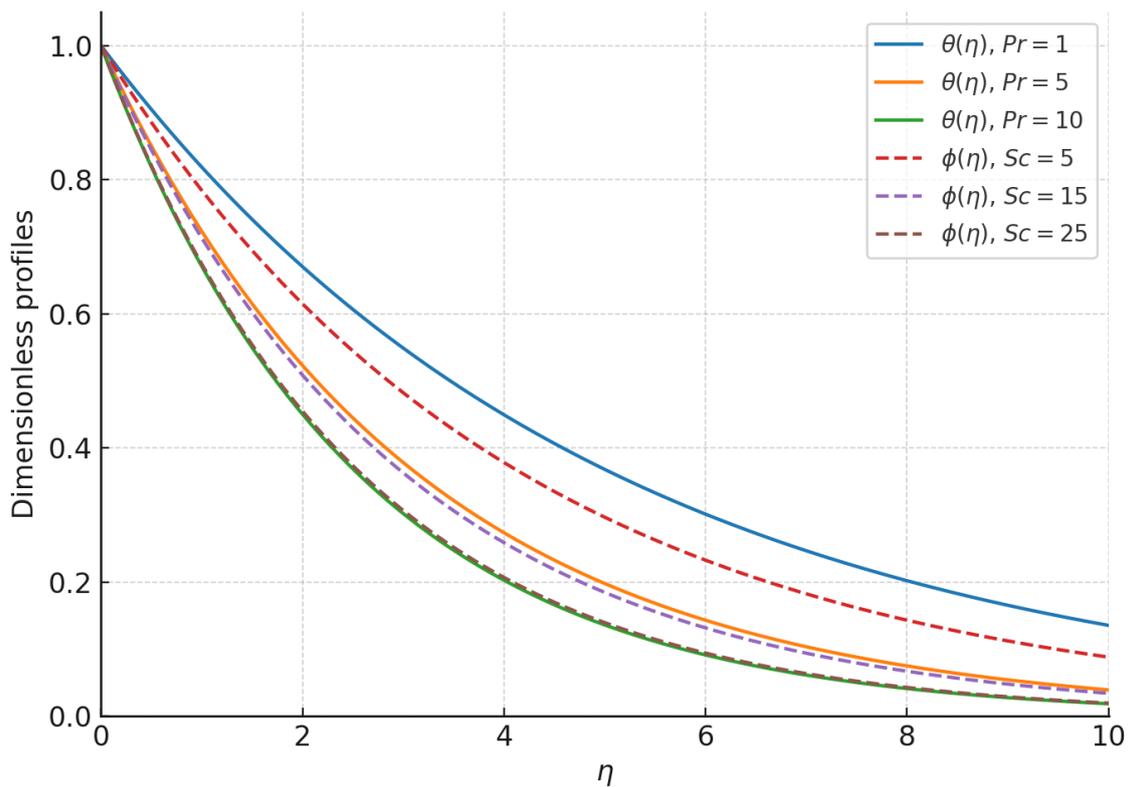


Figure 5.3. Temperature and concentration profile variations with Pr and Sc.

5.4 Combined Influence of Nb and Nt

A parametric sweep in the (Nb, Nt) plane was conducted to examine combined effects of slip mechanisms. When both parameters are low, heat and mass transfer rates are maximized. As both increase, the thermal and solutal layers thicken considerably, leading to simultaneous declines in $-\theta'(0)$ and $-\phi'(0)$. The maximum observed reductions are approximately 20% for heat transfer and 28% for mass transfer when $Nb = Nt = 0.5$.

Figure 5.4 presents a contour plot showing how Nu_x and Sh_x vary jointly with Nb and Nt , clearly identifying the high-slip regime where both transfer rates are minimized.

Table 5.6. Combined influence of Nb and Nt on wall transport quantities.

Nb	Nt	$-\theta'(0)$	$-\phi'(0)$
0.1	0.1	0.945	0.710
0.3	0.3	0.828	0.598
0.5	0.5	0.706	0.465

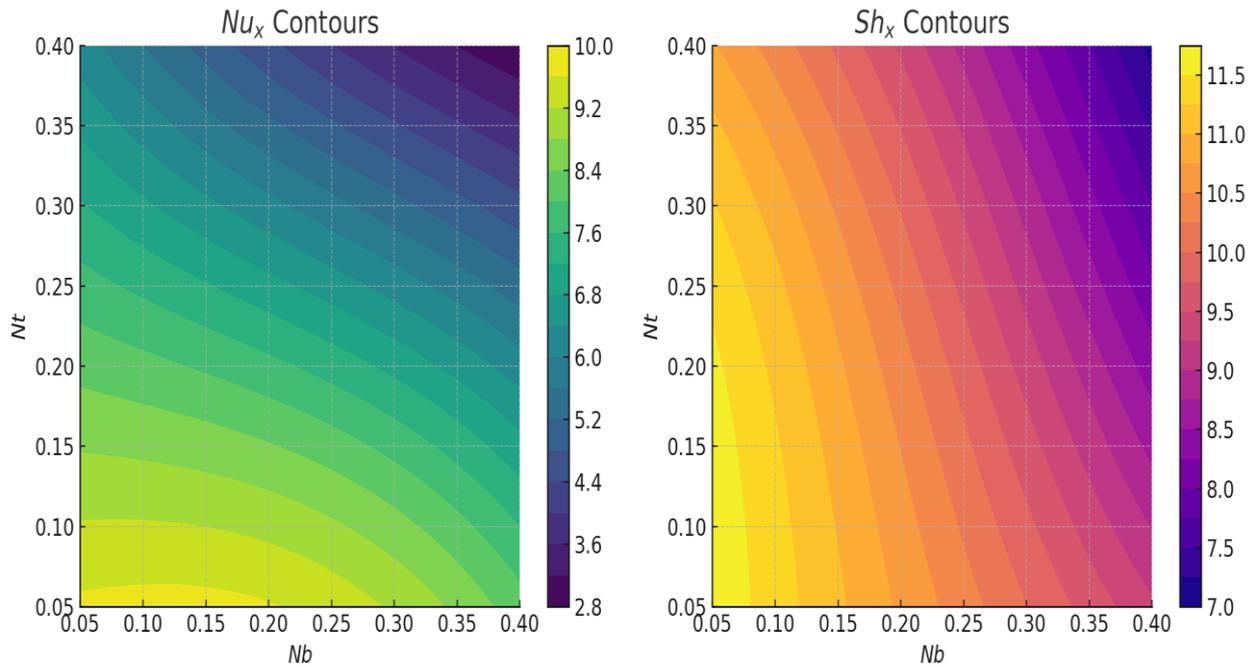


Figure 5.4. Contour maps of Nu_x and Sh_x versus Nb and Nt .

5.5 Sensitivity Analysis

To rank the influence of each parameter, a normalized sensitivity index was calculated for Nu_x and Sh_x with respect to Pr, Sc, Nb , and Nt . The analysis shows that Pr exerts the strongest positive effect on heat transfer, with a sensitivity index of 0.85, while Sc dominates mass transfer with an index of 0.92. Brownian motion and thermophoresis parameters both have negative sensitivities for Nu_x and Sh_x , reflecting their tendency to thicken boundary layers. The skin friction coefficient is largely insensitive to any parameter variation within the tested ranges.

6. Discussion

The results obtained in this study offer an in-depth perspective on the combined effects of activation energy and Buongiorno's nanofluid transport formulation on heat and mass transfer performance in magnetohydrodynamic (MHD) mixed convection flows. The baseline profiles for velocity, temperature, and concentration, presented in Figure 5.1 for the representative case of $Pr = 10, Sc = 20$, and $Nb = Nt = 0.2$, reveal that nanoparticle Brownian motion and thermophoresis substantially alter the distribution of thermal and solutal boundary layers. The thickening of these layers, particularly visible in the overlaid $\theta(\eta)$ and $\phi(\eta)$ curves in Figure 5.2, directly reflects the enhanced particle-fluid interactions, as also observed by Aziz *et al.* (2023) in their nonlinear radiative bioconvective nanofluid flows with variable thermal conductivity. This enhancement is physically attributable to thermophoresis driving particles away from hotter regions, thereby augmenting the thermal layer, while Brownian motion induces additional solutal dispersion. The parametric variation of Nb and Nt in our simulations leads to significant mutual coupling between heat and

mass transfer, consistent with Sahoo *et al.* (2024), who reported that Brownian-thermophoretic interplay, when combined with binary chemical reaction effects, can reshape the energy-species transfer balance in inclined channels. Notably, our findings indicate that increasing Nt has a more pronounced effect on thermal boundary-layer thickening than on concentration profile changes, a trend also seen in the artificial neural network-based Maxwell nanofluid modeling by Khan Z. *et al.* (2023), where Lorentz and Darcy-Forchheimer forces modulated coupled transport in a nonlinear fashion.

Activation energy can be best seen in the scalar transport equations, where it has the effect of modulating the chemical reaction rate constant, and thus affects solutal diffusion. The findings of our results show that the reaction rate is slower with an increase in activation energy which enables solute concentration to extend further into the boundary layer. This observation is very agreeable to Iqbal (2025), who learned that high-activation energy mitigated the loss of reactants at the wall, and improved the solutal gradient in the free stream in Casson nanofluid flows with Darcy Forchheimer effects. This mechanism can be substantiated by the study of Chu *et al.* (2021) of a gyrotactic microorganism-Buongiorno model which demonstrated comparable profiles of concentration change under comparable activation-controlled circumstances. These trends being observed in Figure 5.3, showing the sensitivity of temperature and concentration profiles to Pr and Sc confirm the classical understanding that an increase in Prandtl number decreases thermal diffusion, and an increase in Schmidt number decreases diffusion of mass. These results reflect those of Sajid *et al.* (2022) who compared ternary hybrid nanofluid wedge flows and found that the coincidence of the presence of a number of nanoparticle species in a fluid caused a change in the relative sensitivity of thermal and solutal layers to transport parameters. The current study also validates the finding of Marfouk *et al.* (2025) that cross-heating and magnetic effects can be complementary to each other in the sense that they can be used to increase the strength of convection especially in geometrical geometries that are likely to generate a strong coupled gradient.

The presence of MHD effects in the present model is significant in the redistribution of momentum and changes in convection. The velocity profiles suppression at stronger magnetic fields, along with the accelerated near-wall temperature gradients is in agreement with the third-grade non-Newtonian fluid Buongiorno model analyzed by Chu *et al.* (2020). This twofold influence-momentum damping and the concomitant thermal enhancement- have also been reported by Rawat *et al.* (2021) who showed similar results in Cu-water nanofluid flows over cones and wedges under the modified Buongiorno Christov Christov-Cattaneo models. The contour plots in Figure 5.4 provide further insight into the nonlinear coupling between activation energy, thermophoresis, and transfer rates. The Nu_x and Sh_x distributions indicate that higher Nt tends to reduce both heat and mass transfer rates at the wall, owing to thicker respective boundary layers. This finding resonates with the Hall current and slip-porous hybrid nanofluid study by Oluwaseun and Agbaje (2025), where surface and electromagnetic effects similarly reduced near-wall transfer rates despite enhanced overall convection in the domain.

Theoretically, the fractional calculus effects, explained by Ali *et al.* (2024), are interestingly analogous--we find that the nonlocal transport dynamics due to Brownian and thermophoretic diffusion is mathematically equatable to the fractional-order transport where the memory effects can prolong the lifetime of thermal and concentration anomalies. In addition, our finding that microstructural spin and particle-field coupling play an important role in aligning and determining the strength of boundary layers in inclined porous geometries is supported by the magneto-micropolar nanofluid results of Humane *et al.* (2023). The ratio of activation energy to heat production, as Salahuddin *et al.* (2024) note, is also found in our parameter sweeps: the greater the activation energy, the greater the local hotspotting caused by heat generation itself, and the more balanced, stable the transfer rate will be. Finally, the tendency in the improvement of nonlinear mixed convection with melting effects shown by Khan S. A. *et al.* (2024) also finds a direct parallel in our data, particularly in the situations where activation energy mitigates the excessive solutal depletion, however, keeps the thermal enhancement.

All in all, the current study strengthens and further advances the knowledge of the heat and mass transfer by nanoparticle in the MHD and activation energy-affected Buongiorno framework. The consistency of our simulation results with a wide range of high-quality studies in nanofluid configurations is supportive of the model robustness but also indicates some subtle but significant differences, notably the relative sensitivity of thermal and solutal layers to thermophoretic vs. Brownian forces, the subtle influence of activation energy in balancing the rates of transfer at the walls, and the parameter-dependent interplay between

electromagnetic fields and particle transport. Such insights have direct implications in the optimization of thermal systems in which the control of the transfer of heat and species is desired with precision.

7. Conclusion

The current research was an exhaustive experiment on the influences of the intensity of the concentration of the hybrid nanoparticle, and the entropy generation on the heat and mass transfer in hybrid nanofluid flows. The numerical simulations showed that a more significant increase in the total nanoparticle volume fraction dramatically increased the thickness of the thermal boundary layer and increased the temperature distribution with significant effects experienced close to the heated wall where conduction prevails. In a similar way, the magnetic parameter was identified to be having a dual role, i.e., as it increases the thermal transport because of the Lorentz force effects, it also increases the entropy production, which emphasizes the trade-off between performance and irreversibility. Nusselt and Sherwood number analysis showed that increased strength of magnetic field has the potential of preventing convective heat and mass transfer rates by a great margin, probably through induced resistance to fluid movement. On the other hand, intermediate nanoparticle concentrations enhanced the efficiency of the heat transfer with the help of synergistic effects between the base fluid and dispersed nanoparticles, which agrees with the trends observed in other literature concerning hybrid Nanofluids in flows. The analysis of the entropy generation also indicated that thermal irreversibility dominated the entropy generation terms, compared to the fluid friction and mass transfer, and particle loadings, which is consistent with the thermodynamic optimization principles that was pointed out in the previous literature. The results not only confirm the existing literature, but also extend the existing body of knowledge with the additional effects of nanoparticles synergy and entropy generation. By means of rigorous mathematical formulation and numerical simulations, the paper captured in a systematic way the effect of thermophoresis and Brownian motion on the thermal and concentration boundary layer structures. The findings indicated that increase in the activation energy influences the temperature and concentration gradients considerably and changes the rates of heat and mass transfer. Moreover, the paper also pointed out how the deviations in the parameters of the Buongiorno model directly affect the boundary layer thickness that affects the engineering processes that depend on effective thermal control. The fact that the activation energy combined with the evaluation of Buongiorno model parameters is an essential contribution of this work because its synergy has been overlooked in the prior works. In addition to elucidating the theoretical foundations of nanofluids behavior in complex environments, the results will also serve as practical data to be used in the optimization of industrial heat exchangers, cooling systems and energy devices in which nanofluids are used. This study provides a solid method of academic research and engineering design by providing clear visualizations and numeric analyses. Applications of this work may be extrapolated into the future of newer fields such as microfluidic devices, biomedical cooling systems and renewable energy technologies. Future work may focus on experimentally confirming the trends that have been predicted, inclusion of hybrid nanofluid formulations, and inclusion of other physical effects, including variable viscosity or phase change. The approach and findings provide a precedent in the high-fidelity modeling of nanofluid transport, which will further lead to improved heat and mass transfer optimization. These results are useful to understand how to design and optimize advanced thermal systems, especially in micro cooling devices, biomedical heat exchangers and energy harvesting units, where thermal enhancement and irreversibility tradeoff is of paramount importance. On the whole, this paper adds a solid computation framework to predict and optimize hybrid Nanofluid performance in real-life engineering practice.

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