Stability of Dynamica Systems with Time-Varying Delays

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***Abstract*—This paper is concerned with the asymptotic stability analysis of a class of systems with time-varying delays. New delay-dependent stability criteria are derived in terms of Linear Matrix Inequalities (LMIs), by choosing a new class of Lyapunov-Krasovskii functional (LKFs). A numerical example is given to illustrate the effectiveness of the proposed method.**

Keywords*— Delayed systems, Linear Matrix Inequality (LMI), Lyapunov method, Robust H_∞ control, Stability Analysis, Time-varying delay

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I. INTRODUCTION

Time delays are frequently encountered in many practical engineering systems, such as chemical processes, long transmission lines in pneumatic systems [1]-[8]. It has been shown that the presence of a time delay in a dynamical system is often a primary source of instability and performance degradation [9]. Delay-dependent robust stability criteria of uncertain fuzzy systems with state and input delays are presented in [10]. Dynamical systems with distributed time-varying delays have been of considerable interest for the fast few decades. In particular, the interest in stability analysis of various delay differential systems has been growing rapidly due to their successful applications in practical fields such as circuit theory, aircraft stabilization, population dynamics, distributed networks, manual control and so on. Current efforts on the problem of stability of distributed time-varying delays system can be divided into two categories, namely delay independent criteria and delay dependent criteria. Distributed delay systems have been considered in [11]-[14].

The issue of robust asymptotic stability for Delay-dependent for systems with Time-varying and Distributed delays using Linear Matrix Inequalities (LMI) approach is remains open, which motivates this paper. In this paper, we establish a new LMI condition by using the Lyapunov-Krasovskii functional to guarantee the asymptotic stability of the system. A sufficient condition for the solvability of this problem is proposed in terms of Linear Matrix Inequalities (LMIs). Particularly, the maximal allowable length of delays is obtained from LMI and the validity of this result is checked numerically using the effective LMI control toolbox in MATLAB [15].

NOTATIONS: Throughout this paper, for a matrix B and two symmetric matrices A and C,

$\begin{bmatrix} A & B \\ & C \\ & \end{bmatrix}$ denote the symmetric matrix, where the notation * represents the entries implied by symmetry. A^T and A^{-1} are denotes the matrix transpose and inverse of A respectively. We say $X>0$ for $X \in \Re^n$ means that the matrix X is real symmetric positive definite. $\|\cdot\|$ refers to the Euclidean norm for vectors. And I denotes the identity matrix with appropriate dimensions.

II. SYSTEM DESCRIPTION AND PRELIMINARIES

The following system with time-varying is considered in this paper,

$$\dot{x}(t) = A(t)x(t) + B(t)x(t-\tau(t)) + C(t)\int_{t-d(t)}^{t} x(s)ds \quad (1)$$

$$x(s) = \varphi(s), s \in [-\tau, 0]. \quad (2)$$

where $x(t) \in \Re^n$ is the state. The initial vector $\phi \in C_0$,where C_0 is the set of continuous functions from $[-\tau, 0]$ to $\Re^n$. $\tau(t)$ and $d(t)$ denotes the time-varying and distributed delays respectively, and are is assumed to satisfy.

$$0 \le \tau(t) \le \tau, \quad \dot{\tau}(t) \le u, \quad 0 \le d(t) \le d \quad (3)$$

where τ, d and u are constants. The matrices $A(t) = A + \Delta A(t), B(t) = B + \Delta B(t), C(t) = C + \Delta C(t) and D(t) = D + \Delta D(t)$ are known real constant matrices with appropriate dimensions. $\Delta A(t), \Delta B(t), \Delta C(t)$ and $\Delta D(t)$ are real-valued unknown matrices representing time-varying parameter uncertainties, and are assumed to be of the form.

$$\begin{bmatrix} \Delta A(t) & \Delta B(t) & \Delta C(t) & \Delta D(t) \\ & & & \end{bmatrix} = LF(t)\begin{bmatrix} E_1 & E_2 & E_3 & E_4 \\ & & & \end{bmatrix} \quad (4)$$

Where L and $E_i, (i = 1,2,3,4)$ are known real constant matrices and $F(t)$ is unknown time-varying matrix functions satisfying $F^T(t)F(t) \le 1, \forall \quad t$

Lemma 2.1 (Schur complement [16]). Let M,P,Q be given matrices such that $Q>0$, then

$$\begin{bmatrix} P & M^T \\ M & -Q \end{bmatrix} < 0 \quad \Leftrightarrow P+M^TQ^{-1}M<0.$$

Lemma 2.2 Given any matrices X,Y and S with appropriate dimensions such that $0<S=S^T$,the following inequality holds

$$2X^TY \le X^TSX + Y^TS^{-1}Y.$$

Lemma 2.3 [17] For any constant matrix $M \in R^{n\times n}, M=M^T>0,$ scalar $\eta>0,$ vector function $w:[0,\eta]\to R^n$ such that the integrations concerned are well defined, then

$$\left[\int_0^{\eta} w(s)ds\right]^T M\left[\int_0^{\eta} w(s)ds\right] \le \eta\int_0^{\eta} w^T(s)Mw(s)ds.$$

Lemma 2.4 ([18])For given matrices D,E and F with $F^TF\le I$ and positive scalar $\varepsilon>0,$ the following inequality holds:

$$DFE+E^TF^TD^T \le \varepsilon DD^T + \varepsilon^{-1}E^TE.$$

Lemma 2.5 ([19]) For real matrices $P>0, M=i(i=1,2,3)$ with appropriate dimensions, and $\tau(t)$ satisfying (2), then

$$\int_{t-\tau(t)}^{t} \dot{x}^T(t)P\dot{x}(s)ds \le \xi^T(t)[\tau M^TP^{-1}M + M^T\bar{I}+\bar{I}M]\xi(t)$$

Where,

$$\xi^T(t)=\begin{bmatrix} x^T(t) & x^T(t-\tau(t)) & (\int_{t-d(t)}^{t} x(s)ds)^T & w^T(t) \end{bmatrix}$$

$$M=\begin{bmatrix} M_1 & M_2 & M_3 & 0 \end{bmatrix}, \bar{I}=\begin{bmatrix} I & -I & 0 & 0 \end{bmatrix}$$

III. Main Results

Theorem 3.1 Given scalars $\tau>0, d>0$ and $u>0$, the system described in (1) with time-varying and distributed delays satisfying (2) is asymptotically stable. If there exist matrices $Q>0,$ $R>0$ and appropriately dimensioned matrices $M_l,(l=1,2,3)$, and scalar $\varepsilon>0$ such that the following LMI holds.

$$\begin{bmatrix} \Theta_{11} & \Theta_{12} & \Theta_{13} & \tau PA^T & \tau M_1^T & E_1^T+E_4^T \\ & \Theta_{22} & \Theta_{23} & \tau PB^T & \tau M_2^T & XE_2^T \\ & * & \Theta_{33} & \tau PC^T & \tau M_3^T & XE_3^T \\ & * & * & -\tau P & 0 & 0 \\ & * & * & * & -\tau X & 0 \\ & * & * & * & * & -\varepsilon I \end{bmatrix} \quad (5)$$

Where

$\Theta_{11}=AP+PA^T+Q+dR+M_1+M_1^T+\varepsilon LL^T, \Theta_{12} = PB+M_2-M_1^T,$

$\Theta_{13}=PC+M_3, \Theta_{22}=-(1-u)Q-M_2-M_2^T, \Theta_{23} =-M_3, \Theta_{33}=-\frac{1}{d}R.$

Proof : Define the Lyapunov functional candidate as

$$V(x_t)=x^T(t)Px(t)+\int_{t-\tau(t)}^{t} x^T(s)Qx(s)ds + \int_{-\tau(t)}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)P\dot{x}(s)ds + \int_{-d(t)}^{0}\int_{t+\theta}^{t} x^T(s)Rx(s)ds \quad (6)$$

$$\dot{V}(x_t)=2x^T(t)P\dot{x}(t)+x^T(t)Qx(t) -(1-\dot{\tau}(t))x^T(t-\tau(t))Qx^T(t-\tau(t)) +\tau(t)\dot{x}^T(t)P\dot{x}(t) -\int_{t-\tau(t)}^{t}\dot{x}^T(s)P\dot{x}(s)ds+d(t)x^T(t)Rx(t) -\int_{t-d(t)}^{t} x^T(s)Rx(s)ds \quad (7)$$

Applying Lemma 2.3 and 2.5, we obtain

$$\dot{V}(x_t)=2x^T(t)P[(A(t) +B(t)x(t-\tau(t))+C(t)\int_{t-d(t)}^{t}x(s)ds] +x^T(t)Qx(t)-(1-u)x^T(t-\tau(t)) Qx^T(t-\tau(t))+\tau(t)\dot{x}^T(t)P\dot{x}(t) +\xi^T(t)[M^TP^{-1}M+M^T\bar{I}+\bar{I}^TM]\xi(t) +d(t)x^T(t)Rx(t) -(\int_{t-d(t)}^{t}x(s)ds)^TR(\int_{t-d(t)}^{t}x(s)ds) \quad (8)$$

By using Eqn.(3) and Lemma 2.4, we obtain

$$2x^T(t)P[(\Delta A(t)+\Delta B(t)x(t-\tau(t))$$
$$+\Delta C(t)\int_{t-d(t)}^{t}x(s)ds]$$
$$=2PLF(t)\begin{bmatrix}E_1 & E_2 & E_3\end{bmatrix}\xi(t)$$
$$\le \varepsilon PLL^T P+\varepsilon^{-1}\begin{bmatrix}E_1 & E_2 & E_3\end{bmatrix}^T\begin{bmatrix}E_1 & E_2 & E_3\end{bmatrix} \quad (9)$$

Combining (7) to (9), we have

$$\dot{V}(x_t)=\xi^T(t)[\Sigma_1+\tau M^T P^{-1}M$$
$$+\tau\Sigma_2^T P\Sigma_2+\varepsilon^{-1}\Sigma_3^T\Sigma_3]\xi(t) \quad (10)$$

Where

$$\Sigma_1=\begin{bmatrix}\Sigma_{11} & \Sigma_{12} & \Sigma_{13}\\ & \Sigma_{22} & \Sigma_{23}\\ & * & \Sigma_{33}\end{bmatrix},\Sigma_2^T=\begin{bmatrix}A^T\\ B^T\\ C^T\end{bmatrix},$$
$$\Sigma_3^T=\begin{bmatrix}E_1^T\\ E_2^T\\ E_3^T\end{bmatrix}$$

With

$$\Sigma_{11}=PA+A^T P+Q+dR+M_1$$
$$+M_1^T,\Sigma_{12}=PB+M_2-M_1^T,$$
$$\Sigma_{13}=PC+M_3,\Sigma_{22}=-(1-u)Q-M_2$$
$$-M_2^T,\Sigma_{23}=-M_3,\Sigma_{33}=-\frac{1}{d}R$$

Thus, we conclude that the system (1) with (2) is robustly asymptotically stable.

IV. Numerical Examples

Example: Consider the system (1), with following example matrices,

$$A=\begin{bmatrix}-0.9 & 0.2\\ 0.1 & -0.9\end{bmatrix},\ B=\begin{bmatrix}-1.1 & -0.2\\ -0.1 & -1.1\end{bmatrix},$$
$$C=\begin{bmatrix}0.2 & 0.1\\ 0.1 & 0.2\end{bmatrix},\ D=\begin{bmatrix}-0.12 & -0.12\\ -0.12 & 0.12\end{bmatrix},$$
$$B_w=\begin{bmatrix}0.2 & -0.1\\ 0.1 & 0.2\end{bmatrix},G=\begin{bmatrix}0.5 & 0.2\\ 0.2 & 0.1\end{bmatrix},$$
$$H=\begin{bmatrix}0.5 & -0.2\\ -0.2 & 0.1\end{bmatrix},$$
$$L=0.2I,E_1=E_2=E_3=E_4=0.1I$$

For example the prescribed H_∞ performance level is chosen as $\gamma=1$. In order to design a Delay-dependent H_∞, taking $\tau=0.5,\ \ ,d=0.5$ and $u=0.1$, applying the Theorem 3.1 the LMI solutions are,

$$X=\begin{bmatrix}2.4918 & -0.1563\\ -0.1563 & 2.6731\end{bmatrix},$$
$$\overline{Q}=\begin{bmatrix}2.2885 & -0.1652\\ -0.1652 & 2.3844\end{bmatrix},$$
$$\overline{R}=\begin{bmatrix}1.7496 & -0.0359\\ -0.0359 & 1.7828\end{bmatrix},$$
$$\varepsilon=2.8029$$

By theorem 3.1, we can obtain the desire state feedback controller as follows:

$$K=\begin{bmatrix}0.8056 & 0.0471\\ 0.0471 & 0.7510\end{bmatrix}.$$

Therefore, the concerned system is robustly asymptotically stable.

Conclusion

In this work, we have studied the H_∞ control for uncertain systems with time-varying and distributed delays. On the basis of Lyapunov-Krasovskii functional, a delay-dependent H_∞ control scheme is presented in terms of Linear Matrix Inequality (LMI). It has been shown that a desired state feedback controller can be constructed when the given LMIs are feasible. A numerical example have been carried out to demonstrate the effectiveness and the merit of the proposed method.

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