# Delay-dependent stability criteria for delay differential systems 

C. Jayakumar<br>Assistant professor<br>Mahendra arts and Science College<br>Namakkal, India

G.Manoharan<br>Scholar<br>Mahendra arts and Science College<br>Namakkal, India


#### Abstract

This paper proposes an approach for the stability of delay differential systems. The key features of the approach is that to obtain generalized stability region, a parameterized model transformation with free weighting matrices is introduced. In fact, these techniques lead to generalized and less conservative stability condition that guarantee the wide stability region. The proposed stability conditions are demonstrated with numerical examples. Comparisons with other stability conditions in the literature shows the derived conditions are the more powerful ones to guarantee the widest stability region.


Keywords- Delay differential Systems, Global asymptotic stability; Time-varying delays; Linear matrix inequality; Neural networks

2000 AMS Subject Classification - 34K20, 34K50, 92B20, 94D05

## I. Introduction

Time delays are frequently encountered in many practical engineering systems, such as chemical processes, long transmission lines in pneumatic systems [1]-[8]. It has been shown that the presence of a time delay in a dynamical system is often a primary source of instability and performance degradation [9]. Delay-dependent robust stability criteria of uncertain fuzzy systems with state and input delays are presented in [10]. Dynamical systems with distributed timevarying delays have been of considerable interest for the fast few decades. In particular, the interest in stability analysis of various delay differential systems has been growing rapidly due to their successful applications in practical fields such as circuit theory, aircraft stabilization, population dynamics, distributed networks, manual control and so on. Current efforts on the problem of stability of distributed time-varying delays system can be divided into two categories, namely delay independent criteria and delay dependent criteria. Distributed delay systems have been considered in [11]-[14].

The issue of robust asymptotic stability for delay differential systems using Linear Matrix Inequalities (LMI) approach is remains open, which motivates this paper. In this paper, we establish a new LMI condition by using the Lyapunov-Krasovskii functional to guarantee the asymptotic stability of the system concerned. A sufficient condition for the solvability of this problem is proposed in terms of Linear Matrix Inequalities (LMIs) and the validity of this result is checked numerically using the effective LMI control toolbox in MATLAB.

NOTATIONS: Throughout this paper, for a matrix $B$ and two symmetric matrices $A$ and $C$,

$$
\left[\begin{array}{ll}
A & B \\
& C
\end{array}\right] \text { denote the symmetric matrix, where the notation }
$$

* represents the entries implied by symmetry. $A^{T}$ and $A^{-1}$ are denotes the matrix transpose and inverse of $A$ respectively. We say $X>0$ for $X \in \mathfrak{R}^{n}$ means that the matrix $X$ is real symmetric positive definite. $\mathrm{P} \cdot \mathrm{P}$ refers to the Euclidean norm for vectors. And $I$ denotes the identity matrix with appropriate dimensions.


## II. Main Results

Consider the following delay differential system

$$
\begin{equation*}
\dot{x}(t)=-A x(t)+B f(x(t))+C f(x(t-\tau(t))) \tag{1}
\end{equation*}
$$

Where $\quad x(t)=\left[x_{1}(t), x_{2}(t), \ldots, x_{n}(t)\right]^{T} \in R^{n} \quad$ is the neural state vector. The matrices $A=\operatorname{diag}\left\{a_{1}, a_{2}, \ldots, a_{n}\right\}$ is a diagonal matrix and $a_{i}>0, i=1, \ldots, n, B=\left[b_{i j}\right]_{n \times n}, C=\left[c_{i j}\right]_{n \times n} \quad$ are the connection weight matrices. Further $f(x(t))=\left[f_{1}\left(x_{1}(t)\right), f_{2}\left(x_{2}(t)\right), \ldots, f_{n}\left(x_{n}(t)\right)\right]^{T} \in R^{n}$ is the neuron activation function with $f(0)=0 . \tau(t)$ is the time-varying delay which satisfies

$$
0 \leq \tau_{1} \leq \tau(t) \leq \tau_{2} \text { and } \dot{\tau}(t) \leq d, \text { forevery } t \geq 0
$$

Where $\tau_{1}, \tau_{2}$ and $d$ are known constants.
The following assumption is made on the neuron activation function.
(A) Each neuron activation functions $g(\cdot)$ in system (1) are bounded and satisfy the following condition

$$
0 \leq \frac{g_{i}(x)-g_{i}(y)}{x-y} \leq L_{i}
$$

Where $L_{i}(i=1,2 \ldots, n)$ are some constants and they can be positive. So it is less restrictive than the descriptions on both the sigmoid activations and the Lipschitz type activation functions. Denote $L=\operatorname{diag}\left\{L_{1}, \ldots, L_{n}\right\}$.

Note that the function $f(\cdot)$ satisfies Assumption (A), and we have

$$
f^{T}(x(t)) f(x(t)) \leq x^{T}(t) L^{T} L x(t)
$$

Lemma 2.1 [15] For any vectors $x, y \in R^{n}$, with $P>0$, the following inequality holds

$$
2 x^{T} y \leq x^{T} P^{-1} x+y^{T} P y
$$

## III. Global stability results

In this section, some sufficient conditions of stability for system (1) are obtained.

Theorem 3.1 Given scalars $\tau_{1}>0, \tau_{2}>0$ and $d>0$, if there exist symmetric positive definite matrices $P>0, Q_{j}>0, S_{i}>0,(i=1,2,3),(j=1, \ldots, 4)$, and any matrices $\quad N_{a}, M_{a}, O_{a}, N_{i}, M_{i}, O_{i},(i=1,2, \ldots, 8) \quad$ such that feasible solution exist for the following LMI.

$$
\Omega=\left[\begin{array}{ccccccccccc}
\Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & \Omega_{15} & \Omega_{16} & \Omega_{17} & \Omega_{18} & \Omega_{19} & \Omega_{1,10} & \Omega_{1,11}  \tag{2}\\
* & \Omega_{22} & \Omega_{23} & \Omega_{24} & \Omega_{25} & \Omega_{26} & \Omega_{27} & \Omega_{28} & \Omega_{29} & \Omega_{2,10} & \Omega_{2,11} \\
* & * & \Omega_{33} & \Omega_{34} & \Omega_{35} & \Omega_{36} & \Omega_{37} & \Omega_{38} & \Omega_{39} & \Omega_{3,10} & \Omega_{3,11} \\
* & * & * & \Omega_{44} & \Omega_{45} & \Omega_{46} & \Omega_{47} & \Omega_{48} & \Omega_{49} & \Omega_{4,10} & \Omega_{4,11} \\
* & * & * & * & \Omega_{55} & \Omega_{56} & \Omega_{57} & \Omega_{58} & \Omega_{59} & \Omega_{5,10} & \Omega_{5,11} \\
* & * & * & * & * & \Omega_{66} & \Omega_{67} & \Omega_{68} & \Omega_{69} & \Omega_{6,10} & \Omega_{6,11} \\
* & * & * & * & * & * & \Omega_{77} & \Omega_{78} & \Omega_{79} & \Omega_{7,10} & \Omega_{7,11} \\
* & * & * & * & * & * & * & \Omega_{88} & \Omega_{89} & \Omega_{8,10} & \Omega_{8,11} \\
* & * & * & * & * & * & * & * & \Omega_{99} & \Omega_{9,10} & \Omega_{9,11} \\
* & * & * & * & * & * & * & * & * & \Omega_{10,10} & \Omega_{10,11} \\
* & * & * & * & * & * & * & * & * & * & \Omega_{11,11}
\end{array}\right]<
$$

$$
\begin{aligned}
& \Omega_{11}=\Xi_{0}-O_{1} A-A^{T} O_{1}+O_{1} B L+L^{T} B^{T} O_{1}^{T} \\
& \Omega_{12}=O_{1} C L-A^{T} O_{2}^{T}+L^{T} B^{T} O_{2}^{T}+P C L-M_{1}, \\
& \Omega_{13}=-A^{T} N_{a}^{T}+L^{T} B^{T} N_{a}^{T}+M_{1}, \Omega_{14}=-N_{1}-A^{T} O_{3}^{T}+L^{T} B^{T} O_{3}^{T}, \\
& \Omega_{15}=N_{1}-A^{T} O_{4}^{T}+L^{T} B^{T} O_{4}^{T}, \Omega_{16}=-A^{T} O_{5}^{T}+L^{T} B^{T} O_{5}^{T}, \\
& \Omega_{17}=-A^{T} O_{6}^{T}+L^{T} B^{T} O_{6}^{T}, \\
& \Omega_{18}=-A^{T} O_{7}^{T}+L^{T} B^{T} O_{7}^{T}, \\
& \Omega_{19}=-O_{1}-A^{T} O_{8}^{T}+L^{T} B^{T} O_{8}^{T}, \Omega_{1,10}=N_{1}, \Omega_{1,11}=M_{1}, \\
& \Omega_{22}=-(1-d) Q_{1}+O_{2} C L+L^{T} C^{T} O_{2}^{T}+X-M_{2}^{T}-M_{2}, \Omega_{23}=L^{T} C^{T} O_{a}^{T}+M_{2}-M_{a}^{T}-X, \\
& \Omega_{24}=-N_{2}+L^{T} C^{T} O_{3}^{T}-M_{3}^{T}, \Omega_{25}=N_{2}+L^{T} C^{T} O_{4}^{T}-M_{4}^{T}, \Omega_{26}=L^{T} C^{T} O_{5}^{T}-M_{5}^{T}, \\
& \Omega_{27}=L^{T} C^{T} O_{6}^{T}-M_{6}^{T}, \Omega_{28}=L^{T} C^{T} O_{7}^{T}-M_{7}^{T}, \\
& \Omega_{29}=-O_{2}+L^{T} C^{T} O_{8}^{T}-M_{8}^{T}, \Omega_{2,10}=N_{2}, \Omega_{2,11}=M_{2}, \Omega_{33}=M_{a}+M_{a}^{T}+X-Q_{4}, \\
& \Omega_{34}=-N_{a}+M_{3}^{T}, \Omega_{35}=M_{a}+M_{4}^{T}, \Omega_{36}=-U_{22}^{T}+M_{5}^{T}, \Omega_{37}=M_{6}^{T}, \Omega_{38}=M_{7}^{T},
\end{aligned}
$$

$$
\begin{aligned}
& \Omega_{39}=-O_{a}+M_{8}^{T}, \Omega_{3,10}=N_{a}, \Omega_{3,11}=M_{a}, \Omega_{4,4}=-Q_{2}-N_{3}^{T}-N_{3}+W, \Omega_{4,5}=N_{3}-N_{4}^{T}-W, \\
& \Omega_{4,6}=-N_{5}^{T}, \Omega_{4,7}=-V_{22}^{T}-N_{6}^{T}, \Omega_{4,8}=-N_{7}^{T}, \Omega_{4,9}=-O_{3}-N_{8}^{T}, \Omega_{4,10}=N_{3}, \Omega_{4,11}=M_{3}, \\
& \Omega_{5,5}=N_{4}^{T}+N_{4}+W-Q_{3}, \Omega_{5,6}=N_{5}^{T}, \Omega_{5,7}=N_{6}^{T}, \Omega_{5,8}=N_{7}^{T}, \Omega_{5,9}=-O_{4}+N_{8}^{T} \\
& \Omega_{5,10}=N_{4}, \Omega_{5,11}=M_{4}, \Omega_{6,6}=-S_{1}, \Omega_{6,7}=0, \Omega_{6,8}=0, \Omega_{6,9}=-O_{5}, \Omega_{6,10}=N_{5}, \Omega_{6,11}=M_{5}, \\
& \Omega_{7,7}=-S_{2}, \Omega_{7,8}=0, \Omega_{7,9}=-O_{6}, \Omega_{7,10}=N_{6}, \Omega_{7,11}=M_{6}, \Omega_{8,8}=-S_{3}, \Omega_{8,9}=-O_{7}, \\
& \Omega_{8,10}=N_{7}, \Omega_{8,11}=M_{7}, \Omega_{9,9}=-O_{8}^{T}-O_{8}, \Omega_{9,10}=N_{8}, \Omega_{9,11}=M_{8}, \Omega_{10,10}=-W, \Omega_{10,11}=0, \\
& \Omega_{11,11}=-X, \Xi_{0}=-A^{T} P-P A+Q_{1}+Q_{2}+Q_{3}+Q_{4}+\tau_{a}^{2} S_{1}+\tau_{1}^{2} S_{2}+\tau_{2}^{2} S_{3}+P B L+L^{T} B^{T} P,
\end{aligned}
$$

$\tau_{a}=\frac{1}{2}\left(\tau_{1}+\tau_{2}\right)$, for then system (1) is asymptotically
Proof: We use the following Lyapunov functional to derive the stability result.
stable.

$$
\begin{equation*}
V(t, x(t))=V_{1}(t, x(t))+V_{2}(t, x(t))+V_{3}(t, x(t))+V_{4}(t, x(t))+V_{5}(t, x(t))+V_{6}(t, x(t)) \tag{3}
\end{equation*}
$$

Where

$$
V_{1}(t, x(t))=x^{T}(t) P x(t)
$$

$$
\begin{gathered}
V_{2}(t, x(t))=\int_{t-\tau(t)}^{t} x^{T}(s) Q_{1} x(s) d s+\int_{t-\tau_{1}}^{t} x^{T}(s) Q_{2} x(s) d s+\int_{t-\tau_{2}}^{t} x^{T}(s) Q_{3} x(s) d s+\int_{t-\tau_{a}}^{t} x^{T}(s) Q_{4} x(s) d s \\
V_{3}(t, x(t))=\tau_{a} \int_{-\tau_{a}}^{0} \int_{t+\beta}^{t} x^{T}(s) S_{1} x(s) d s d \beta+\tau_{1} \int_{-\tau_{1}}^{0} \int_{t+\beta}^{t} x^{T}(s) S_{2} x(s) d s d \beta+\tau_{2} \int_{-\tau_{2}}^{0} \int_{t+\beta}^{t} x^{T}(s) S_{3} x(s) d s d \beta \\
V_{4}(t, x(t))=\left[\begin{array}{c}
x(t) \\
\int_{t-\tau_{a}}^{t} x(s) d s
\end{array}\right]^{T}\left[\begin{array}{cc}
U_{11} & U_{12} \\
* & U_{22}
\end{array}\right]\left[\begin{array}{c}
x(t) \\
\int_{t-\tau_{a}}^{t} x(s) d s
\end{array}\right], \\
V_{5}(t, x(t))=\left[\begin{array}{c}
x(t) \\
\int_{t-\tau_{1}}^{t} x(s) d s
\end{array}\right]^{T}\left[\begin{array}{cc}
V_{11} & V_{12} \\
* & V_{22}
\end{array}\right]\left[\begin{array}{c}
x(t) \\
\int_{t-\tau_{1}}^{t} x(s) d s
\end{array}\right] \\
V_{6}(t, x(t))=\left[\begin{array}{ll}
x & x(t) \\
\int_{t-\tau_{2}}^{t} x(s) d s
\end{array}\right]^{T}\left[\begin{array}{ll}
W_{11} & W_{12} \\
* & W_{22}
\end{array}\right]\left[\begin{array}{l}
x(t) \\
\int_{t-\tau_{2}}^{t} x(s) d s
\end{array}\right]
\end{gathered}
$$

We can calculate derivative of $V(t)$ along the trajectories of the system (1), then we have .

$$
\begin{align*}
& \dot{V}_{1}(t, x(t)) \leq 2 x^{T}(t) P[-A x(t)+B f(x(t))+C f(x(t-\tau(t)))]  \tag{4}\\
& \dot{V}_{2}(t, x(t)) \leq x^{T}(t) Q_{1} x(t)-(1-\dot{\tau}(t)) x^{T}(t-\tau(t)) Q_{1} x(t-\tau(t))+x^{T}(t) Q_{2} x(t)+x^{T}(t) Q_{3} x(t) \\
& \quad+x^{T}(t) Q_{4} x(t)-x^{T}\left(t-\tau_{1}\right) Q_{2} x\left(t-\tau_{1}\right)-x^{T}\left(t-\tau_{2}\right) Q_{3} x\left(t-\tau_{2}\right)-x^{T}\left(t-\tau_{a}\right) Q_{4} x\left(t-\tau_{a}\right) \tag{5}
\end{align*}
$$

$$
\begin{align*}
& \dot{V}_{3}(t, x(t)) \leq \tau_{a}^{2} x^{T}(t) S_{1} x(t)+\tau_{1}^{2} x^{T}(t) S_{2} x(t)+\tau_{2}^{2} x^{T}(t) S_{3} x(t) \\
& -\tau_{a} \int_{t-\tau_{a}}^{t} x^{T}(s) S_{1} x(s) d s-\tau_{1} \int_{t-\tau_{1}}^{t} x^{T}(s) S_{2} x(s) d s-\tau_{2} \int_{t-\tau_{2}}^{t} x^{T}(s) S_{3} x(s) d s \tag{6}
\end{align*}
$$

Thus from, (4)-(7) and using Jensen's inequality [?] in (7)

$$
\dot{V}(t, x(t)) \leq \xi_{4}^{T}(t)\left[\begin{array}{cccccccc}
(1,1) & (1,2) & \Xi_{3} & \Xi_{13} & \Xi_{23} & \Xi_{4} & \Xi_{14} & \Xi_{24} \\
* & (2,2) & \Xi_{6} & \Xi_{16} & \Xi_{26} & \Xi_{7} & \Xi_{17} & \Xi_{27} \\
* & * & (3,3) & 0 & 0 & \Xi_{9} & 0 & 0 \\
* & * & * & (4,4) & 0 & 0 & \Xi_{19} & 0 \\
* & * & * & * & (5,5) & 0 & 0 & \Xi_{29} \\
* & * & * & * & * & (6,6) & 0 & 0 \\
* & * & * & * & * & * & (7,7) & 0 \\
* & * & * & * & * & * & * & (8,8)
\end{array}\right] \xi_{4}(t),
$$

Where

$$
\begin{gathered}
(1,1)=\Xi_{0}+\Xi_{1}+\Xi_{11}+\Xi_{21}, \quad(1,2)=P C L+\Xi_{2}+\Xi_{12}+\Xi_{22}, \\
(2,2)=-(1-d) Q_{1}+\Xi_{5}+\Xi_{15}+\Xi_{25},(3,3)=-Q_{4}+\Xi_{8},(4,4)=-Q_{2}+\Xi_{18}, \\
(5,5)=-Q_{3}+\Xi_{28},(6,6)=-S_{1}+\Xi_{10},(7,7)=-S_{2}+\Xi_{20},(8,8)=-S_{3}+\Xi_{30} \\
\xi_{4}^{T}(t)=\left[x^{T}(t) x^{T}(t-\tau(t)) x^{T}\left(t-\tau_{a}\right) x^{T}\left(t-\tau_{1}\right) x^{T}\left(t-\tau_{2}\right)\left(\int_{t-\tau_{a}}^{t} x(s) d s\right)^{T}\left(\int_{t-\tau_{1}}^{t} x(s) d s\right)^{T}\left(\int_{t-\tau_{2}}^{t} x(s) d s\right)^{T}\right] .
\end{gathered}
$$

According to Leibniz-Newton formula, for any matrices
$N_{a}, N_{i},(i=1, \ldots, 8)$, the following equation holds

$$
\begin{align*}
& 2\left[x^{T}(t) N_{1}+x^{T}(t-\tau(t)) N_{2}+x^{T}\left(t-\tau_{1}\right) N_{3}+x^{T}\left(t-\tau_{2}\right) N_{4}+\left(\int_{t-\tau_{a}}^{t} x(s) d s\right)^{T} N_{5}\right. \\
& \left.+\left(\int_{t-\tau_{1}}^{t} x(s) d s\right)^{T} N_{6}+\left(\int_{t-\tau_{2}}^{t} x(s) d s\right)^{T} N_{7}+\dot{x}^{T}(t) N_{8}+x^{T}\left(t-\tau_{a}\right) N_{a}\right] \\
& \times \tag{7}
\end{align*}
$$

It follows from Lemma 2.1 and by Leibniz-Newton formula that

$$
\begin{aligned}
-2\left[x^{T}(t) N_{1}+\right. & x^{T}(t-\tau(t)) N_{2}+x^{T}\left(t-\tau_{1}\right) N_{3}+x^{T}\left(t-\tau_{2}\right) N_{4}+\left(\int_{t-\tau_{a}}^{t} x(s) d s\right)^{T} N_{5} \\
& \left.+\left(\int_{t-\tau_{1}}^{t} x(s) d s\right)^{T} N_{6}+\left(\int_{t-\tau_{2}}^{t} x(s) d s\right)^{T} N_{7}+\dot{x}^{T}(t) N_{8}+x^{T}\left(t-\tau_{a}\right) N_{a}\right] \int_{t-\tau_{1}}^{t-\tau_{2}} \dot{x}(s) d s \\
\leq & \xi^{T}(t) N W^{-1} N^{T} \xi(t)+\left(\int_{t-\tau_{1}}^{t-\tau_{2}} \dot{x}(s) d s\right)^{T} W\left(\int_{t-\tau_{1}}^{t-\tau_{2}} \dot{x}(s) d s\right)
\end{aligned}
$$

$$
\begin{equation*}
\leq \xi^{T}(t) N W^{-1} N^{T} \xi(t)+\left[x\left(t-\tau_{2}\right)-x\left(t-\tau_{1}\right)\right]^{T} W\left[x\left(t-\tau_{2}\right)-x\left(t-\tau_{1}\right)\right] \tag{8}
\end{equation*}
$$

Where

$$
\begin{gathered}
\xi^{T}(t)=\left[x^{T}(t) x^{T}(t-\tau(t)) x^{T}\left(t-\tau_{a}\right) x^{T}\left(t-\tau_{1}\right) x^{T}\left(t-\tau_{2}\right)\left(\int_{t-\tau_{a}}^{t} x(s) d s\right)^{T}\left(\int_{t-\tau_{1}}^{t} x(s) d s\right)^{T}\left(\int_{t-\tau_{2}}^{t} x(s) d s\right)^{T} \dot{x}^{T}(t)\right. \\
a n d N^{T}=\left[N_{1}^{T} N_{2}^{T} N_{a}^{T} N_{3}^{T} N_{4}^{T} N_{5}^{T} N_{6}^{T} N_{7}^{T} N_{8}^{T}\right]
\end{gathered}
$$

According to Leibniz-Newton formula, for any matrices
$M_{a}, M_{i},(i=1, \ldots, 8)$, the following equation holds

$$
\begin{align*}
& 2\left[x^{T}(t) M_{1}+x^{T}(t-\tau(t)) M_{2}+\right. x^{T}\left(t-\tau_{1}\right) M_{3}+x^{T}\left(t-\tau_{2}\right) M_{4}+\left(\int_{t-\tau_{a}}^{t} x(s) d s\right)^{T} M_{5} \\
&\left.+\left(\int_{t-\tau_{1}}^{t} x(s) d s\right)^{T} M_{6}+\left(\int_{t-\tau_{2}}^{t} x(s) d s\right)^{T} M_{7}+\dot{x}^{T}(t) M_{8}+x^{T}\left(t-\tau_{a}\right) M_{a}\right] \\
& \times\left[x\left(t-\tau_{a}\right)-x(t-\tau(t))-\int_{t-\tau(t)}^{t-\tau} \dot{x}(s) d s\right]=0 \tag{9}
\end{align*}
$$

It follows from Lemma 2.1 and by Leibniz-Newton formula that

$$
\begin{aligned}
-2 & {\left[x^{T}(t) M_{1}+x^{T}(t-\tau(t)) M_{2}+x^{T}\left(t-\tau_{1}\right) M_{3}+x^{T}\left(t-\tau_{2}\right) M_{4}+\left(\int_{t-\tau_{a}}^{t} x(s) d s\right)^{T} M_{5}\right.} \\
& \left.+\left(\int_{t-\tau_{1}}^{t} x(s) d s\right)^{T} M_{6}+\left(\int_{t-\tau_{2}}^{t} x(s) d s\right)^{T} M_{7}+\dot{x}^{T}(t) M_{8}+x^{T}\left(t-\tau_{a}\right) M_{a}\right] \int_{t-\tau(t)}^{t-\tau} a \dot{x}(s) d s \\
\leq & \xi^{T}(t) N W^{-1} N^{T} \xi(t)+\left(\int_{t-\tau_{1}}^{t-\tau_{2}} \dot{x}(s) d s\right)^{T} W\left(\int_{t-\tau(t)}^{t-\tau_{a}} \dot{x}(s) d s\right)
\end{aligned}
$$

$$
\begin{equation*}
\leq \xi^{T}(t) M X^{-1} M^{T} \xi(t)+\left[x\left(t-\tau_{a}\right)-x(t-\tau(t))\right]^{T} X\left[x\left(t-\tau_{a}\right)-x(t-\tau(t))\right] \tag{10}
\end{equation*}
$$

Where
$M^{T}=\left[M_{1}^{T} M_{2}^{T} M_{a}^{T} M_{3}^{T} M_{4}^{T} M_{5}^{T} M_{6}^{T} M_{7}^{T} M_{8}^{T}\right]$.

According to equation (3), for any matrices $O_{a}, O_{i},(i=1, \ldots, 8)$, the following equation holds

$$
\begin{align*}
2\left[x^{T}(t)\right. & O_{1}+x^{T}(t-\tau(t)) O_{2}+x^{T}\left(t-\tau_{1}\right) O_{3}+x^{T}\left(t-\tau_{2}\right) O_{4}+\left(\int_{t-\tau_{a}}^{t} x(s) d s\right)^{T} O_{5} \\
+ & \left.\left(\int_{t-\tau_{1}}^{t} x(s) d s\right)^{T} O_{6}+\left(\int_{t-\tau_{2}}^{t} x(s) d s\right)^{T} O_{7}+\dot{x}^{T}(t) O_{8}+x^{T}\left(t-\tau_{a}\right) O_{a}\right] \\
& \times[-\dot{x}(t)+\{-A x(t)+B f(x(t))+C f(x(t-\tau(t)))\}]=0 \tag{11}
\end{align*}
$$

Then, we add the terms on the left sides of equation (8), (10), (12) to $\dot{V}(t)$ and consider the equation (9), (11), we obtain

$$
\dot{V}(t) \leq \xi^{T}(t) \Omega \xi(t)
$$

This completes the proof.

## IV. Numerical Examples

Example 1. Consider the system (1) with time varying delays. The model of this system is of the following form:

$$
\dot{x}(t)=-A x(t)+B f(x(t))+W f(x(t-\tau(t)))
$$

and

$$
\begin{aligned}
& A=\left[\begin{array}{ll}
7 & 0 \\
0 & 5
\end{array}\right], \quad B=\left[\begin{array}{cc}
-0.3 & -0.2 \\
0.3 & 0.4
\end{array}\right], \\
& C=\left[\begin{array}{ll}
-0.5 & 0.7 \\
-0.8 & -1
\end{array}\right] \quad L=\left[\begin{array}{ll}
1 & 0 \\
0 & 1
\end{array}\right] .
\end{aligned}
$$

$$
S_{3}=\left[\begin{array}{ll}
0.7435 & 0.0025 \\
0.0025 & 0.6968
\end{array}\right]
$$

Therefore, the concerned system with time-varying delays is asymptotically stable.

## Conclusion

A new sufficient condition is derived to guarantee the stability of the equilibrium point for fuzzy cellular neural networks with interval time varying delays. A linear matrix inequality approach has been developed to solve the problem addressed. Our results can be easily verified and also less conservative than previously known criteria.

By using the Matlab LMI toolbox, we solve the LMI (3) for $\tau_{1}=0.5, \tau_{2}=1.5$ and $d=0.5$ the feasible solutions are

$$
\begin{aligned}
& P=\left[\begin{array}{ll}
0.6632 & 0.0127 \\
0.0127 & 0.5652
\end{array}\right], \\
& Q_{1}=\left[\begin{array}{ll}
0.9233 & 0.0815 \\
0.0815 & 1.3429
\end{array}\right], \\
& Q_{2}=\left[\begin{array}{ll}
0.8747 & 0.0017 \\
0.0017 & 0.8365
\end{array}\right], \\
& Q_{3}=\left[\begin{array}{ll}
0.8742 & 0.0013 \\
0.0013 & 0.8374
\end{array}\right], \\
& Q_{4}=\left[\begin{array}{ll}
0.9669 & 0.0036 \\
0.0036 & 0.9270
\end{array}\right], \\
& S_{1}=\left[\begin{array}{ll}
0.7560 & 0.0019 \\
0.0019 & 0.7387
\end{array}\right], \\
& S_{2}=\left[\begin{array}{ll}
0.7494 & 0.0007 \\
0.0007 & 0.7477
\end{array}\right],
\end{aligned}
$$

## REFERENCES

[1] Liu PL, Delay-dependent stabilization for linear time-delay uncertain systems with saturating actuators, Int. J. Gen. Syst. 40 (2011) 301 - 312.
[2] Liu PL, Robust stability for neutral time-varying delay systems with nonlinear perturbations, International journal of Innovative Computing, Information and Control 10 (7) (2011) 5749 - 5760.
[3] Fridman E, Shaked U, A descriptor system approach to control of linear time-delay systems, IEEE Trans. Automat. control 47 (2002) 253 - 270.
[4] Gao HJ, Wang CH, Delay-dependent robust and filtering for a class of uncertain nonlinear time-delay systems, IEEE Trans. Automat. Control 48 (9) (2003) 1661-1666.
[5] Peng C, Han QL, Yue D, Tian E, Sampled-data robust control for T-S fuzzy systems with time delay and uncertainties, Fuzzy Sets and Systems 179 (2011) $20-33$.
[6] Peng C, Tian YC, Delay-dependent robust control for uncertain systems with time-varying delay. Inform. Sci. 179 (2009) 3187 - 3197.
[7] Wang JW, Wu HN, Guo L, Luo YS, Robust fuzzy control for uncertain nonlinear Markovian jump systems with time-varying delay, Fuzzy Sets and Systems 212 (2013) 41-61.
[8] Zhang H, Shen Y, Feng G, Delay-dependent stability and control for a class of fuzzy descriptor systems with time-delay, Fuzzy Sets and Systems 160 (2009) 1689-1707.
[9] Li X, Souza CED, Delay-dependent robust stability and stabilization of uncertain linear delay systems: a linear matrix inequality approach, IEEE Trans. Automat. Control 42 (1997) 1144-1148.
[10] Li L, Liu X, New results on delay-dependent robust stability criteria of uncertain fuzzy systems with state and input delays, Inform. Sci. 179 (2009) 1134-1148.
[11] Chen H, Zhang Y, Zhao Y, Stability analysis for uncertain neutral systems with discrete and distributed delays, Appl. Math. Comput. 218 (2012) 11351-11361.
[12] Li XG, Zhu XJ, Stability analysis of netural systems with distributed delays, Automatica (2008) $1-5$.
[13] Song X, Xu S, Shen H, Robust control for uncertain fuzzy systems with distributed delays via output feedback controllers.Inform. Sci. 178 (2008) 4341 - 4356.
[14] Xie L, Fridman E, Shaked U, Robust control of distributed delay systems with application to the combustion control, IEEE Trans. Automat. Control 46 (2001) 1930-1935.
[15] Y. Wang, L. Xie, C. E. de souza, Robust control of a class of uncertain nonlinear systems, Systems Control Lett, vol. 19, 139-149, 1992.

